Mathematical Studies

for the IB DIPLOMA

Second Edition

for 2012 syllabus

WORKED SOLUTIONS

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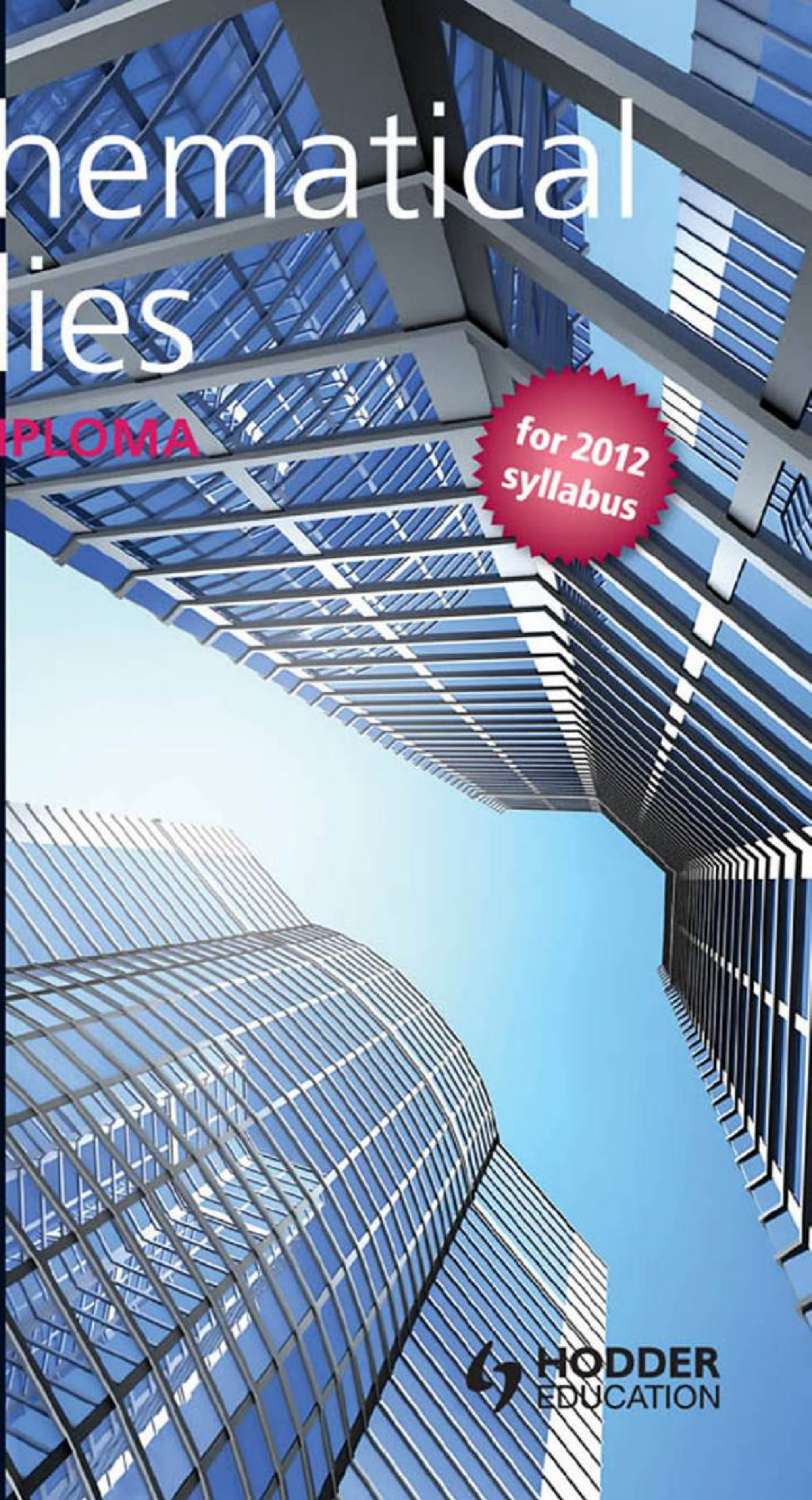
FREE ONLINE RESOURCES



HODDER EDUCATION

Worked solutions

Worked solutions have been provided for all exercises and assessments from the textbook except for questions that do not require working or where the working has already been provided in the answer section of the textbook.

Exercise 1.2.4

1 **a** $60 \times 20 = 1200$
b $300 \times 10 = 3000$
c $50 \times 60 = 3000$
d $5000 \times 30 = 150\,000$
e $0.8 \times 1 = 0.8$
f $0.2 \times 500 = 100$

2 **a** $4000 \div 20 = 200$
b $8000 \div 40 = 200$
c $900 \div 30 = 30$
d $5500 \div 10 = 550$
e $50 \div 0.1 = 500$
f $600 \div 0.2 = 3000$

3 **a** $80 + 50 = 130$
b $170 - 90 = 80$
c $3 \div 3 = 1$
d $80 \div 20 = 4$
e $\dfrac{4 \times 800}{16} = \dfrac{3200}{16} = 200$
f $\dfrac{10^3}{2^2} = \dfrac{1000}{4} = 250$

5 **a** $20\,\text{m} \times 6\,\text{m} = 120\,\text{m}^2$
b $(10\,\text{m} \times 5\,\text{m}) - (3\,\text{m} \times 2\,\text{m})$
$\approx 50 - 10 = 40\,\text{m}^2$
c $(30\,\text{cm} \times 15\,\text{cm}) - (10\,\text{cm} \times 4\,\text{cm})$
$\approx 450 - 50 = 400\,\text{cm}^2$

6 **a** $10\,\text{cm} \times 10\,\text{cm} \times 2\,\text{cm} = 200\,\text{cm}^3$
b $20\,\text{cm} \times 40\,\text{cm} \times 5\,\text{cm} = 4000\,\text{cm}^3$
c $(20\,\text{cm} \times 5\,\text{cm} \times 10\,\text{cm}) + ((24 - 4)\,\text{cm} \times 10\,\text{cm} \times 5\,\text{cm}) = 2000\,\text{cm}^3$

Exercise 1.2.5

1 **a** 984 is 980 to two significant figures.
Percentage error $= \frac{4}{984} \times 100 = 0.4\%$
b 2450 is 2500 to two significant figures.
Percentage error $= \frac{50}{2450} \times 100 = 2.04\%$
c 504 is 500 to two significant figures.
Percentage error $= \frac{4}{504} \times 100 = 0.8\%$

2 First player's percentage error is
$\dfrac{250 - 240}{240} \times 100 = \dfrac{10}{240} \times 100 = 4.2\%$
Second player's percentage error is
$\dfrac{258 - 250}{258} \times 100 = \dfrac{8}{258} \times 100 = 3.1\%$
So the second player had the smaller percentage error.

3 **a** The maximum possible height is
$9500 \times 1.025 = 9737.5\,\text{m}$
b The minimum possible height is
$9500 \times 0.975 = 9262.5\,\text{m}$

4 **a** Actual speed is $\dfrac{120}{1.015} = 118.2\,\text{km}\,\text{h}^{-1}$
b Percentage error is
$\dfrac{180 - 175}{175} \times 100 = \dfrac{5}{175} \times 100 = 2.9\%$

Exercise 1.3.1

4 **a** $200 \times 3000 = 2 \times 10^2 \times 3 \times 10^3$
$= 6 \times 10^5$
b $6000 \times 4000 = 6 \times 10^3 \times 4 \times 10^3$
$= 24 \times 10^6$
$= 2.4 \times 10^7$
c $7 \text{ million} \times 20 = 7 \times 10^6 \times 2 \times 10^1$
$= 14 \times 10^7$
$= 1.4 \times 10^8$
d $500 \times 6 \text{ million} = 5 \times 10^2 \times 6 \times 10^6$
$= 30 \times 10^8$
$= 3 \times 10^9$
e $3 \text{ million} \times 4 \text{ million} = 3 \times 10^6 \times 4 \times 10^6$
$= 12 \times 10^{12}$
$= 1.2 \times 10^{13}$
f $4500 \times 4000 = 4.5 \times 10^3 \times 4 \times 10^3$
$= 18 \times 10^6$
$= 1.8 \times 10^7$

5 Distance = speed × time = $3 \times 10^8 \times 8 \times 60 = 144 \times 10^9 = 1.44 \times 10^{11}$ m

6 **a** $(4.4 \times 10^3) \times (2 \times 10^5) = (4.4 \times 2) \times 10^{5+3}$
$= 8.8 \times 10^8$

b $(6.8 \times 10^7) \times (3 \times 10^3) = (6.8 \times 3) \times 10^{7+3}$
$= 20.4 \times 10^{10}$
$= 2.04 \times 10^{11}$

c $(4 \times 10^5) \times (8.3 \times 10^5) = (4 \times 8.3) \times 10^{5+5}$
$= 33.2 \times 10^{10}$
$= 3.32 \times 10^{11}$

d $(5 \times 10^9) \times (8.4 \times 10^{12}) = (5 \times 8.4) \times 10^{9+12}$
$= 42 \times 10^{21}$
$= 4.2 \times 10^{22}$

e $(8.5 \times 10^6) \times (6 \times 10^{15}) = (8.5 \times 6) \times 10^{6+15}$
$= 51 \times 10^{21}$
$= 5.1 \times 10^{22}$

f $(5.0 \times 10^{12})^2 = (5.0 \times 10^{12}) \times (5.0 \times 10^{12})$
$= (5 \times 5) \times 10^{12+12}$
$= 25 \times 10^{24} = 2.5 \times 10^{25}$

7 **a** $(3.8 \times 10^8) \div (1.9 \times 10^6)$
$= (3.8 \div 1.9) \times 10^{8-6} = 2 \times 10^2$

b $(6.75 \times 10^9) \div (2.25 \times 10^4)$
$= (6.75 \div 2.25) \times 10^{9-4} = 3 \times 10^5$

c $(9.6 \times 10^{11}) \div (2.4 \times 10^5)$
$= (9.6 \div 2.4) \times 10^{11-5} = 4 \times 10^6$

d $\dfrac{1.8 \times 10^{12}}{9.0 \times 10^7} = (1.8 \div 9) \times 10^{12-7}$
$= 0.2 \times 10^5$
$= 2 \times 10^4$

e $\dfrac{2.3 \times 10^{11}}{9.2 \times 10^4} = (2.3 \div 9.2) \times 10^{11-4}$
$= 0.25 \times 10^7$
$= 2.5 \times 10^6$

f $\dfrac{2.4 \times 10^8}{6 \times 10^3} = (2.4 \div 6) \times 10^{8-3}$
$= 0.4 \times 10^5$
$= 4 \times 10^4$

8 **a** $(3.8 \times 10^5) + (4.6 \times 10^4)$
$= (38 \times 10^4) + (4.6 \times 10^4)$
$= 42.6 \times 10^4$
$= 4.26 \times 10^5$

b $(7.9 \times 10^9) + (5.8 \times 10^8)$
$= (79 \times 10^8) + (5.8 \times 10^8)$
$= 84.8 \times 10^8$
$= 8.48 \times 10^9$

c $(6.3 \times 10^7) + (8.8 \times 10^5)$
$= (630 \times 10^5) + (8.8 \times 10^5)$
$= 638.8 \times 10^5$
$= 6.388 \times 10^7$

d $(3.15 \times 10^9) + (7.0 \times 10^6)$
$= (3150 \times 10^6) + (7.0 \times 10^6)$
$= 3157 \times 10^6$
$= 3.157 \times 10^9$

e $(5.3 \times 10^8) - (8.0 \times 10^7)$
$= (53 \times 10^7) - (8.0 \times 10^7)$
$= 45 \times 10^7$
$= 4.5 \times 10^8$

f $(6.5 \times 10^7) - (4.9 \times 10^6)$
$= (65 \times 10^6) - (4.9 \times 10^6)$
$= 60.1 \times 10^6$
$= 6.01 \times 10^7$

g $(8.93 \times 10^{10}) - (7.8 \times 10^9)$
$= (89.3 \times 10^9) - (7.8 \times 10^9)$
$= 81.5 \times 10^9$
$= 8.15 \times 10^{10}$

h $(4.07 \times 10^7) - (5.1 \times 10^6)$
$= (40.7 \times 10^6) - (5.1 \times 10^6)$
$= 35.6 \times 10^6$
$= 3.56 \times 10^7$

9

Jupiter	$778 \times 10^6 = 7.78 \times 10^8$ km
Mercury	$58 \times 10^6 = 5.8 \times 10^7$ km
Mars	$228 \times 10^6 = 2.28 \times 10^8$ km
Uranus	$2870 \times 10^6 = 2.87 \times 10^9$ km
Venus	$108 \times 10^6 = 1.08 \times 10^8$ km
Neptune	$4500 \times 10^6 = 4.5 \times 10^9$ km
Earth	$150 \times 10^6 = 1.5 \times 10^8$ km
Saturn	$1430 \times 10^6 = 1.43 \times 10^9$ km

The list in the answers in increasing order of magnitude is the 10^7 term, then 10^8 terms listed by increasing value of a, and then 10^9 terms listed by increasing value of a.

Exercise 1.7.1

1 **a** $u_2 = u_1 + 5 = 3 + 5 = 8$, $u_3 = 8 + 5 = 13$, $u_4 = 13 + 5 = 18$; arithmetic

b $u_2 = 2u_1 - 4 = 2 - 4 = -2$,
$u_3 = 2 \times -2 - 4 = -8$,
$u_4 = 2 \times -8 - 4 = -20$; not arithmetic

c $u_2 = -4u_1 + 1 = 1$, $u_3 = (-4 \times 1) + 1 = -3$,
$u_4 = (-4 \times -3) + 1 = 12 + 1 = 13$;
not arithmetic

d $u_2 = 3 - u_1 = 3 - 5 = -2$, $u_3 = 3 - -2 = 5$,
$u_4 = 3 - 5 = -2$; not arithmetic

e $u_2 = -4 + u_1 = -4 + 8 = 4$, $u_3 = -4 + 4 = 0$,
$u_4 = -4 + 0 = -4$; arithmetic

f $u_2 = 6 - \frac{1}{3}u_1 = 6 + 3 = 9$, $u_3 = 6 - 3 = 3$,
$u_4 = 6 - 1 = 5$; not arithmetic

2 a 5, 8, 11, 14, 17

i Common difference is 3. From first term, $5 = (3 \times 1) + \text{constant}$, i.e. constant = 2. So formula is $u_n = 3n + 2$

ii 10th term, $u_{10} = (3 \times 10) + 2 = 32$

b 0, 4, 8, 12, 16

i Common difference is 4. From first term, $0 = (4 \times 1) + \text{constant}$, i.e. constant = –4. So formula is $u_n = 4n - 4$

ii 10th term, $u_{10} = (4 \times 10) - 4 = 36$

c $\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 4\frac{1}{2}$

i Common difference is 1. From first term, $\frac{1}{2} = (1 \times 1) + \text{constant}$, i.e. constant = –0.5. So formula is $u_n = n - 0.5$

ii 10th term, $u_{10} = 10 - 0.5 = 9.5$

d 6, 3, 0, –3, –6

i Common difference is –3. From first term, $6 = (-3 \times 1) + \text{constant}$, i.e. constant = 9. So formula is $u_n = -3n + 9$

ii 10th term, $u_{10} = (-3 \times 10) + 9 = -21$

e –7, –4, –1, 2, 5

i Common difference is 3. From first term, $-7 = (3 \times 1) + \text{constant}$, i.e. constant = –10. So formula is $u_n = 3n - 10$

ii 10th term, $u_{10} = (3 \times 10) - 10 = 20$

f –9, –13, –17, –21, –25

i Common difference is –4. From first term, $-9 = (-4 \times 1) + \text{constant}$, i.e. constant = –5. So formula is $u_n = -4n - 5$

ii 10th term, $u_{10} = (-4 \times 10) - 5 = -45$

4 a 5, 9, 13, 17, 21

i Common difference is 4

ii From first term, $5 = (4 \times 1) + \text{constant}$, i.e. constant = 1. So $u_n = 4n + 1$

iii $u_{50} = (4 \times 50) + 1 = 201$

b 0, _, 2, _, 4

i Common difference is 1

ii From first term, $0 = (1 \times 1) + \text{constant}$, i.e. constant = –1. So $u_n = n - 1$

iii $u_{50} = (1 \times 50) - 1 = 49$

c –10, _, _, _, 2

i If sequence is arithmetic, $4d = 12$, i.e. common difference is 3

ii From first term, $-10 = (3 \times 1) + \text{constant}$, i.e. constant = –13. So $u_n = 3n - 13$

iii $u_{50} = (3 \times 50) - 13 = 137$

d $u_1 = 6$, $u_9 = 10$

i If sequence is arithmetic, $8d = 4$, i.e. common difference is 0.5

ii From first term, $6 = (0.5 \times 1) + \text{constant}$, i.e. constant = 5.5. So $u_n = 0.5n + 5.5$

iii $u_{50} = (0.5 \times 50) + 5.5 = 30.5$

e $u_3 = -50$, $u_{20} = 18$

i If sequence is arithmetic, $17d = 68$, i.e. common difference is 4

ii From third term, $-50 = (4 \times 3) + \text{constant}$, i.e. constant = –62. So $u_n = 4n - 62$

iii $u_{50} = (4 \times 50) - 62 = 138$

f $u_5 = 60$, $u_{12} = 39$

i If sequence is arithmetic, $7d = -21$, i.e. common difference is –3

ii From fifth term, $60 = (-3 \times 5) + \text{constant}$, i.e. constant = 75. So $u_n = -3n + 75$

iii $u_{50} = (-3 \times 50) + 75 = -75$

5 Bond pays for itself when total interest is \$200. Each year interest is $200 \times 0.125 = \$25$. It will therefore take $(200 \div 25) = 8$ years.

Exercise 1.7.2

2 a Using $S_n = \frac{n}{2}(u_1 + u_n)$,

$$\sum_1^{10}(4 - n) = \frac{10}{2}(3 - 6) = -15$$

b $$\sum_1^{20}\left(\frac{n}{2} - 10\right) = \frac{20}{2}(-9.5 + 0) = -95$$

c $$\sum_{10}^{20}(3n - 50) = \sum_1^{20}(3n - 50) - \sum_1^{9}(3n - 50) = \frac{20}{2}(-47 + 10) - \frac{9}{2}(-47 - 23) = -370 + 315 = -55$$

d $$\sum_1^{n}\left(\frac{n}{2} + 4\right) = \frac{n}{2}\left(\frac{1}{2} + 4 + \frac{n}{2} + 4\right) = \frac{n}{2}\left(\frac{17}{2} + \frac{n}{2}\right) = \frac{n(n + 17)}{4}$$

3 a If $u_2 = -2$, $u_6 = 10$, then $4d = 12$, i.e. $d = 3$

b $u_1 = u_2 - 3 = -2 - 3 = -5$

c $u_n = 3n + \text{constant}$. Looking at u_1, $-5 = 3 \times 1 + \text{constant}$. So constant = –8. Therefore $u_{20} = 3 \times 20 - 8 = 52$

d $S_{20} = \frac{20}{2}(-5 + 52) = 10 \times 47 = 470$

4 a $S_{10} = \frac{10}{2}(u_1 + u_n)$, i.e. $95 = \frac{10}{2}(u_1 + 18.5)$.
So $5u_1 = 95 - 92.5 = 2.5$, i.e. $u_1 = 0.5$

b $u_1 = 0.5$, $u_{10} = 18.5$. So $9d = 18$, i.e. $d = 2$

c Looking at first term, $0.5 = 2 \times 1 + \text{constant}$.
So constant $= -1.5$.
$S_{50} = \frac{50}{2}(0.5 + 100 - 1.5) = 25 \times 99 = 2475$

5 a $d = 9 - m - (m - 9) = m - 9 - (3 - 3m)$.
So $-2m + 18 = 4m - 12$,
i.e. $6m = 30$, giving $m = 5$. So $d = 8$

b $u_5 = 3 - 3m = 3 - 15 = -12$.
To find the constant: $-12 = 8 \times 5 + \text{constant}$,
i.e. constant $= -52$.
So $u_1 = (8 \times 1) - 52 = -44$

c $S_{10} = \frac{10}{2}(u_1 + u_n) = 5(-44 + [(8 \times 10) - 52])$
$= 5(-44 + 28) = 5 \times -16 = -80$

6 a $u_1 = x$, $u_4 = 2x$. So $3d = 2x - x = x$, i.e. $d = \frac{x}{3}$

b If $u_{10} = 24$, $9d = 24 - x$. Therefore from a,
$3x = 24 - x$, giving $x = 6$

c $S_{10} = \frac{10}{2}(6 + 24) = 5 \times 30 = 150$

7 $-176 = \frac{n}{2}(19 - 51)$, i.e. $-176 = -16n$.
So $n = 11$

8 a Student's proof

b $78 = \frac{n}{2}(n + 1)$, i.e. $156 = n^2 + n$.
So $n^2 + n - 156 = 0$,
i.e. $(n + 13)(n - 12) = 0$.
Taking the positive root, the number of rows is 12.

c If $n = 19$, number of bricks required is
$\frac{19}{2} \times 20 = 190$
If $n = 20$, number of bricks required is
$\frac{20}{2} \times 21 = 210$
So the maximum number of rows with 200 bricks is 19.

Exercise 1.8.1

2 a 2, 6, 18, 54

i Common ratio is $6 \div 2 = 3$

ii $54 \times 3 = 162$, $162 \times 3 = 486$

iii Formula is $u_n = u_1 r^{n-1}$. So $u_n = 2(3)^{n-1}$

b 25, 5, 1, $\frac{1}{5}$

i Common ratio is $5 \div 25 = \frac{1}{5}$

ii $\frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$, $\frac{1}{25} \times \frac{1}{5} = \frac{1}{125}$

iii Formula is $u_n = u_1 r^{n-1}$. So $u_n = 25\left(\frac{1}{5}\right)^{n-1}$

d $-3, 9, -27, 81$

i Common ratio is $9 \div -3 = -3$

ii $81 \times -3 = -243$, $-243 \times -3 = 729$

iii Formula is $u_n = u_1 r^{n-1}$. So $u_n = -3(-3)^{n-1}$

3 $u_n = -6 \times 2^{n-1}$

a $u_1 = -6 \times 2^0 = -6$, $u_2 = -6 \times 2^1 = -12$,
$u_3 = -6 \times 2^2 = -24$

b $-768 = -6 \times 2^{n-1}$, i.e. $2^{n-1} = 128 = 2^7$.
So $n - 1 = 7$, i.e. $n = 8$

4 $u_2 = -1$, $u_5 = 64$

a $-1 \times r^3 = 64$, i.e. $r^3 = -64$, giving $r = -4$

b $u_1 = u_2 \div -4$. So $u_1 = -1 \div -4 = \frac{1}{4}$

c Using $u_n = u_1 r^{n-1}$, $u_{10} = \frac{1}{4}(-4)^9 = -65\,536$

Exercise 1.8.2

1 a $\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + 1 + 2$

i $r = \frac{1}{4} \div \frac{1}{8} = 2$

ii Using $S_n = \dfrac{u_1(r^n - 1)}{r - 1}$,
$S_{10} = \frac{1}{8}(2^{10} - 1) \div (2 - 1) = \frac{1023}{8}$

b $-\frac{1}{9} + \frac{1}{3} - 1 + 3 - 9$

i $r = \frac{1}{3} \div -\frac{1}{9} = -3$

ii Using $S_n = \dfrac{u_1(r^n - 1)}{r - 1}$,
$$S_{10} = -\frac{1}{9} \times \frac{(-3 - 1)}{-3 - 1} = \frac{59\,048}{36} = \frac{14\,762}{9}$$

c $5 + 7.5 + 11.25 + 16.875$

i $r = 7.5 \div 5 = 1.5$

ii $S_{10} = 5\dfrac{(1.5^{10} - 1)}{1.5 - 1} = 10(57.665 - 1)$
$= 566.65$

d $10 + 1 + 0.1 + 0.01 + 0.001$

i $r = 1 \div 10 = 0.1$

ii Using $S_n = \dfrac{u_1(1 - r^n)}{1 - r}$,
$$S_{10} = 10\frac{(1 - 0.1^{10})}{1 - 0.1} \cong 10 \div 0.9$$
$= 11.11111111$

2 a $1 + 3 + 9 + \ldots + 2187$

i $u_1 = 1$, $r = 3$. Using $u_n = u_1 r^{n-1}$,
$2187 = 3^{n-1}$, i.e. $3^7 = 3^{n-1}$.
So $7 = n - 1$, i.e. $n = 8$

ii $S_8 = 1\dfrac{(3^8 - 1)}{3 - 1} = 6560 \div 2 = 3280$

b $\frac{1}{5} + \frac{2}{5} + \frac{4}{5} + \ldots + 12\frac{4}{5}$

i $u_1 = \frac{1}{5}$, $r = 2$. Using $u_n = u_1 r^{n-1}$, $\frac{64}{5} = \frac{1}{5}2^{n-1}$,
i.e. $2^6 = 2^{n-1}$. So $6 = n - 1$, $n = 7$

ii $S_7 = \frac{1}{5}\dfrac{(2^7 - 1)}{2 - 1} = \frac{127}{5}$

c $8 - 4 + 2 - \ldots + \frac{1}{32}$

i $u_1 = 8$, $r = -\frac{1}{2}$. Using $u_n = u_1 r^{n-1}$,
$\frac{1}{32} = 8\left(-\frac{1}{2}\right)^{n-1}$, i.e. $\frac{1}{256} = \left(-\frac{1}{2}\right)^{n-1}$.
As $\frac{1}{256} > 0$, n must be odd, giving $\dfrac{1}{2^8} = \dfrac{1}{2^{n-1}}$

, i.e. $8 = n - 1$, $n = 9$

ii $S_9 = 8\dfrac{\left(1 - \left(-\frac{1}{2}\right)^9\right)}{1 + \frac{1}{2}} = 8\left(1 + \frac{1}{512}\right) \times \frac{2}{3}$
$= 8 \times \frac{513}{512} \times \frac{2}{3} = \frac{171}{32}$

d i Number of terms is n

ii $S_n = \dfrac{u_1(r^n - 1)}{r - 1} = \dfrac{a(r^n - 1)}{r - 1}$

3 a $\sum_1^5 4^n = 4 + 4^2 + \ldots + 4^5$
$= \dfrac{4(4^5 - 1)}{4 - 1} = \dfrac{4 \times 1023}{3} = 1364$

b $\sum_1^7 2(3)^{n-2} = \frac{2}{3} + 2 + 6 + \ldots$
$= \dfrac{\frac{2}{3}(3^7 - 1)}{3 - 1} = \frac{2186}{3} = 728\frac{2}{3}$

c $\sum_4^8 \dfrac{2^{n-1}}{4} = \sum_1^8 \dfrac{2^{n-1}}{4} - \sum_1^3 \dfrac{2^{n-1}}{4}$
$= \frac{1}{4}(2^8 - 1) - \frac{1}{4}(2^3 - 1) = \frac{1}{4}(255 - 7)$
$= \frac{248}{4} = 62$

4 $u_2 = \frac{1}{3}$, $u_5 = 72$

a $u_2 \times r^3 = u_5$, i.e. $\frac{1}{3}r^3 = 72$. So $r^3 = 216$,
i.e. $r = 6$

b $u_1 = u_2 \div 6 = \frac{1}{18}$

c $S_6 = \dfrac{\frac{1}{18}(6^6 - 1)}{6 - 1} = \frac{46655}{90} = \frac{9331}{18}$

5 $(p - 2)$, $(-p + 1)$, $(2p - 2)$

a $(p - 2)r = (-p + 1)$ (1)
$(-p + 1)r = (2p - 2)$ (2)
From (1) $r = \dfrac{(1 - p)}{p - 2}$.
Substituting in (2),
$(1 - p)^2 = 2(p - 1)(p - 2)$
$1 - 2p + p^2 = 2p^2 - 6p + 4$
$0 = p^2 - 4p + 3$
$0 = (p - 1)(p - 3)$
So $p = 3$ or $p = 1$

b If $p = 3$ in (1), $r = -3 + 1 = -2$
The term before $(p - 2)$ is $(p - 2) \div -2$
$= 1 \div -2 = -\frac{1}{2}$

c If $u_3 = (p - 2) = 1$
then with $r = -2$, $u_1 = 1 \div (-2)^2 = \frac{1}{4}$
$S_8 = \dfrac{\frac{1}{4}\left((-2)^8 - 1\right)}{-2 - 1} = -\frac{255}{12} = -\frac{85}{4}$

Exercise 1.8.3

1 a $18 + 6 + 2 + \frac{2}{3} + \ldots$
$u_1 = 18$, $r = \frac{1}{3}$
$S_\infty = \dfrac{u_1}{1 - r} = \dfrac{18}{1 - \frac{1}{3}} = 18 \times \frac{3}{2} = 27$

b $-8 + 4 - 2 + 1 - \ldots$
$u_1 = -8$, $r = -\frac{1}{2}$
$S_\infty = \dfrac{u_1}{1 - r} = \dfrac{-8}{1 - -\frac{1}{2}}$
$= -8 \times \frac{2}{3} = -\frac{16}{3}$

c $1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots$
$u_1 = 1$, $r = \frac{1}{10}$
$S_\infty = \dfrac{1}{1 - \frac{1}{10}} = \frac{10}{9}$

d $7 + 2 + \frac{4}{7} + \frac{8}{49} + \ldots$
$u_1 = 7$, $r = \frac{2}{7}$
$S_\infty = \dfrac{u_1}{1 - r} = \dfrac{7}{1 - \frac{2}{7}} = 7 \times \frac{7}{5} = \frac{49}{5}$

2 a $\sum_1^\infty \left(\frac{1}{4}\right)^n = \dfrac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{3}$

b $\sum_1^\infty \dfrac{2}{2^{n-1}} = \dfrac{2}{1 - \frac{1}{2}} = 4$

c $\sum_{5}^{\infty}\left(\frac{2}{3}\right)^n = \sum_{1}^{\infty}\left(\frac{2}{3}\right)^n - \sum_{1}^{4}\left(\frac{2}{3}\right)^n$

$= \frac{\frac{2}{3}}{1-\frac{2}{3}} - \frac{\frac{2}{3}\left(1-\left(\frac{2}{3}\right)^4\right)}{1-\frac{2}{3}}$

$= 2 - 2\left(1 - \frac{16}{81}\right) = \frac{32}{81}$

d $\sum_{10}^{\infty}\frac{5}{3^{n-1}} = \sum_{1}^{\infty}\frac{5}{3^{n-1}} - \sum_{1}^{9}\frac{5}{3^{n-1}}$

$= \frac{5}{1-\frac{1}{3}} - \frac{5\left(1-\left(\frac{1}{3}\right)^9\right)}{1-\frac{1}{3}} = \frac{15}{2}\left(\frac{1}{3}\right)^9$

$= 0.000\,381$

3 $S_\infty = \frac{u_1}{1-r} = \frac{\frac{u_2}{r}}{1-r} = \frac{\frac{3}{2}}{r(1-r)} = 6$

So $4r^2 - 4r + 1 = 0$, i.e., $(2r-1)^2 = 0$,

giving $r = \frac{1}{2}$ and $u_1 = \frac{\frac{3}{2}}{\frac{1}{2}} = 3$

4 $u_1 + u_2 = 12$

If $r = \frac{1}{3}$, $u_1 + \frac{1}{3}u_1 = 12$, i.e. $\frac{4}{3}u_1 = 12$, so $u_1 = 9$

$S_\infty = \frac{u_1}{1-r} = \frac{9}{1-\frac{1}{3}} = \frac{27}{2}$

Exercise 1.9.1

1 Using $I = \frac{Crn}{100}$

a $I = \frac{300 \times 6 \times 4}{100} = \text{NZ\$72}$

b $I = \frac{750 \times 8 \times 7}{100} = £420$

c $I = \frac{425 \times 6 \times 4}{100} = 102¥$

d $I = \frac{2800 \times 4.5 \times 2}{100} = 252$ baht

e $I = \frac{880 \times 6 \times 7}{100} = \text{HK\$369.60}$

2 Using $n = \frac{100I}{Cr}$

a $n = \frac{100 \times 150}{500 \times 6} = 5$ years

b $n = \frac{100 \times 950}{5800 \times 4} = 4$ years

c $n = \frac{100 \times 1500}{4000 \times 7.5} = 5$ years

d $n = \frac{100 \times 1904}{2800 \times 8.5} = 8$ years

e $n = \frac{100 \times 243}{900 \times 4.5} = 6$ years

f $n = \frac{100 \times 252}{400 \times 9} = 7$ years

3 Using $r = \frac{100I}{Cn}$

a $r = \frac{100 \times 1120}{400 \times 4} = 70\%$

b $r = \frac{100 \times 224}{800 \times 7} = 4\%$

c $r = \frac{100 \times 210}{2000 \times 3} = 3.5\%$

d $r = \frac{100 \times 675}{1500 \times 6} = 7.5\%$

e $r = \frac{100 \times 340}{850 \times 5} = 8\%$

f $r = \frac{100 \times 275}{1250 \times 2} = 11\%$

4 Using $C = \frac{100I}{rn}$

a $C = \frac{100 \times 80}{4 \times 5} = 400$ Ft

b $C = \frac{100 \times 36}{6 \times 3} = \text{NZ\$200}$

c $C = \frac{100 \times 340}{5 \times 8} = €850$

d $C = \frac{100 \times 540}{6 \times 7.5} = 1200$ baht

e $C = \frac{100 \times 540}{3 \times 4.5} = €4000$

f $C = \frac{100 \times 348}{4 \times 7.25} = \text{US\$1200}$

5 $r = \frac{100I}{Cn} = \frac{100 \times 400}{2000 \times 5} = 4\%$

6 $n = \frac{100I}{Cr} = \frac{100 \times 56}{350 \times 8} = 2$ years

7 $r = \frac{100I}{Cn} = \frac{100 \times 108}{480 \times 5} = 4.5\%$

8 Interest is $1320 - 750 = €570$

$r = \frac{100I}{Cn} = \frac{100 \times 570}{750 \times 8} = 9.5\%$

9 $I = \frac{Crn}{100} = \frac{1500 \times 3.5 \times 6}{100} = \text{AU\$}315$

10 Interest is 830 − 500 = 330 baht

$r = \frac{100I}{Cn} = \frac{100 \times 330}{500 \times 11} = 6\%$

Exercise 1.9.2

1 Compound interest, $I = C\left(1 + \frac{r}{100}\right)^n - C$

$= 70\left(1 + \frac{5}{100}\right)^3 - 70 = \$11.03375$ million

$= \$11\,033\,750$

2 $I = C\left(1 + \frac{r}{100}\right)^n - C$

$= 100\,000 \times (1.15)^3 - 100\,000 = €52\,087.50$

3 Sum after 4 years $= 5000 \times (1.20)^4 = \$10\,368$

4 Number at the beginning of fourth year is $1000 \times (1.1)^3 = 1331$ students

5 Weight caught after 4 years is $8\,000\,000 \times (0.8)^4 = 3\,276\,800$ tonnes

6 Let initial amount be u_0

After 1 year debt is $1.42u_0$

After 2 years debt is $1.42 \times 1.42u_0 = 2.016u_0$

So it takes 2 years for the debt to double.

7 Let initial amount be u_0

After 1 year debt is $1.15u_0$

After 4 years debt is $(1.15)^4 u_0 = 1.75u_0$

After 5 years debt is $(1.15)^5 u_0 = 2.01u_0$

So it takes 5 years for the debt to double.

8 Let initial value be u_0

Value after 4 years is $(0.85)^4 u_0 = 0.522u_0$

Value after 5 years is $(0.85)^5 u_0 = 0.444u_0$

Value after 4.5 years is $(0.85)^{4.5} u_0 = 0.481u_0$

Value after 4.25 years is $(0.85)^{4.25} u_0 = 0.501u_0$

Value after 4 years and 4 months is $(0.85)^{4.333} u_0 = 0.495u_0$.

So it will take 4 years and 3 months (to the nearest month) for the value to halve.

9 a After 1 year interest is $3600 \times 0.095 = \$342$

In the next 6 months interest is

$3942 \times \frac{0.095}{2} = \$187$

Total interest after 18 months is \$529

b Interest for each half-yearly period is $\frac{9.5}{2}\%$

Interest after 18 months is

$3600 \times (1.0475)^3 - 3600 = \$537.75$

c Interest for each month is $\frac{9.5}{12}\%$

Interest after 18 months is

$3600 \times (1.007916)^{18} - 3600 = \$549.02$

10 a Interest $= 960 \times (1.075)^2 - 960 = €149.40$

b Interest for each 6-monthly period is $\frac{7.5}{2}\%$

Interest after 2 years is

$960 \times (1.0375)^4 - 960 = €152.30$

c Interest for each month is $\frac{7.5}{12}\%$

Interest after 2 years is

$960 \times (1.00625)^{24} - 960 = €154.84$

Student assessment 2 (Topic 1)

5 a $4000 \times 30\,000 = 4 \times 10^3 \times 3 \times 10^4$

$= 12 \times 10^{3+4} = 1.2 \times 10^8$

b $(2.8 \times 10^5) \times (2.0 \times 10^3) = 5.6 \times 10^{5+3}$

$= 5.6 \times 10^8$

c $(3.2 \times 10^9) \div (1.6 \times 10^4) = 2 \times 10^{9-4} = 2 \times 10^5$

d $(2.4 \times 10^8) \div (9.6 \times 10^2) = 0.25 \times 10^{8-2}$

$= 2.5 \times 10^5$

6 Jupiter is 7.78×10^8 km $= 7.78 \times 10^{11}$ m from the Sun.

Time to reach Jupiter is $7.78 \times 10^{11} \div 3 \times 10^8$ seconds, i.e. $\frac{7.78}{3 \times 60} \times 10^{11-8}$ minutes

$= 43.2$ minutes

7 500 light years $= 500 \times 3 \times 10^5 \times 365 \times 24 \times 60 \times 60$ km $= 4.73 \times 10^{15}$ km

8 $162\,000$ km $= 162\,000 \times 10^3$ m

$= 162\,000 \times 10^3$ m $\times 10^3$ mm

$= 1.62 \times 10^5 \times 10^6 = 1.62 \times 10^{11}$ mm

9 $7\,415\,000$ mg $= 7415$ g $= 7.415$ kg $= 7$ kg (to nearest kg)

Student assessment 3 (Topic 1)

4 Using $I = \frac{Crn}{100}$, interest is $\frac{300 \times 7 \times 5}{100} = \$105$.

So total sum after 5 years is \$405.

5 $u_3 = 27$, $u_6 = -1$

a $u_3 \times r^3 = u_6$, i.e. $27r^3 = -1$.

So $r^3 = -\frac{1}{27}$, i.e. $r = -\frac{1}{3}$

b $u_1 = \frac{u_3}{r^2} = \frac{27}{\frac{1}{9}} = 243$

c $u_n = -\frac{1}{81}$
If $u_6 = -1$, $-1 \times \left(-\frac{1}{3}\right)^x = -\frac{1}{81}$ which gives $x = 4$. So $n = 6 + 4 = 10$

6 a Using $S_n = \frac{n}{2}(u_1 + u_n)$, $\sum_1^{10}(4n - 15)$
$= \frac{10}{2}(-11 + 25) = 5 \times 14 = 70$

b $\sum_5^{18} -5n + 100 = \sum_1^{18} -5n + 100 - \sum_1^{4} -5n + 100$
$= 9(95 + 10) - 2(95 + 80)$
$= (9 \times 105) - (2 \times 175) = 595$

7 a If $u_3 = -6$, $u_{10} = 15$, then $7d = 21$, i.e. $d = 3$
b $u_1 = u_3 - 2d$, i.e. $u_1 = -6 - 6 = -12$
c If $u_{10} = 15$, $u_{20} = 15 + 10d = 15 + 30 = 45$
Using $S_n = \frac{n}{2}(u_1 + u_n)$, $S_{20} = 10(-12 + 45)$
$= 10 \times 33 = 330$

8 a $d = (3m + 1) - (2m + 2) = (5m - 5) - (3m + 1)$
So $m - 1 = 2m - 6$, i.e. $m = 5$ and $d = 4$
b $u_1 = u_3 - 2d$ and $u_3 = 2m + 2 = 12$.
So $u_1 = 12 - (2 \times 4) = 4$
c $u_n = 4n$ + constant.
As $u_1 = 4$, constant = 0 so $u_{10} = 40$
$S_{10} = \frac{10}{2}(4 + 40) = 5 \times 44 = 220$

9 a i $10(2^{n-1}) = 10\,240$, i.e. $2^{n-1} = 1024 = 2^{10}$.
So $n - 1 = 10$, i.e. $n = 11$
ii $S_{11} = 10\,\frac{(2^{11} - 1)}{2 - 1} = 10 \times 2047 = 20\,470$
b i $128\left(-\frac{1}{2}\right)^{n-1} = \frac{1}{32}$, i.e. $\left(-\frac{1}{2}\right)^{n-1} = \frac{1}{2^{12}}$.
So $n - 1 = 12$, i.e. $n = 13$
ii $S_{13} = 128\,\frac{\left(1 - \left(-\frac{1}{2}\right)^{13}\right)}{1 - -\frac{1}{2}}$
$= 128\left(1 + \frac{1}{8192}\right) \times \frac{2}{3} = \frac{2731}{32}$

10 a $\sum_1^5 3^n = \frac{3(3^5 - 1)}{3 - 1} = 363$

b $\sum_3^9 \frac{3^{n-2}}{5} = \sum_1^9 \frac{3^{n-2}}{5} - \sum_1^2 \frac{3^{n-2}}{5}$
$= \frac{\frac{3^{-1}}{5}(3^9 - 1)}{3 - 1} - \frac{\frac{3^{-1}}{5}(3^2 - 1)}{3 - 1} = \frac{3279}{5}$

Student assessment 4 (Topic 1)

1 $10 \times (1 + r)^{100} = 40 \times 10^6$,
giving $1 + r = 1.1642$. So $r = 0.1642 \approx 16.4\%$

2 \$1 000 000 = €1 000 000 ÷ 1.35
= £1 000 000 ÷ 1.35 ÷ 1.32 = £561 167

3 Simple interest over 10 years is 10×0.1, i.e. 100% of initial sum
Compound interest is $(1.1)^{10} - 1 = 2.59 - 1 = 1.59$, i.e. 159% of initial sum
So difference is 59%

4 In 3 years population would be
$86\,000 \times (1.05)^3 = 99\,555$
In 4 years population would be
$86\,000 \times (1.05)^4 = 104\,533$
So population would exceed 100 000 for the first time during 2001

5 a Simple interest
$= 3\,000\,000 \times 0.08 \times 2 =$ €480 000
b Compound interest
$= 3\,000\,000 \times (1.08)^2 - 3\,000\,000 =$ €499 200

6 After 3 years house is $(1.2)^3 = 1.728$ times initial value
After 4 years house is $(1.2)^4 = 2.07$ times initial value, i.e. it doubles in value after 4 years.

7 After 7 days population is $(1.1)^7 = 1.95$ times initial population
After 8 days population is $(1.1)^8 = 2.14$ times initial population, i.e. the population doubles after 8 days.

8 a Simple interest
$= 5 \times 0.06 \times 3 =$ €0.9 million = €900 000
b Compound interest
$= 5 \times (1.06)^3 - 5 =$ €0.95508 million
= €955 080
c Compound interest calculated quarterly
$= 5 \times (1.015)^{12} - 5 =$ €978 091

9 After 4 years boat is $(0.85)^4 = 0.52$ times its initial value
After 5 years boat is $(0.85)^5 = 0.44$ times its initial value, i.e. it takes 5 years to halve in value.

Student assessment 5 (Topic 1)

1 $100\,000 \times (1 + r)^{57} = 140\,000\,000$
$(1 + r)^{57} = 1400$, which gives $1 + r = 1.136$, i.e. $r = 0.136$ or 13.6%

2 $£1\,000\,000 = €1\,000\,000 \times 1.32$
$= \$1\,000\,000 \times 1.32 \times 1.35 = \$1\,782\,000$

3 12.5% simple interest for 20 years
$= 0.125 \times 20 = 2.5$ times initial sum
12.5% compound interest for 20 years
$= (1.125)^{20} - 1 = 9.55$ times initial sum
Percentage difference is 955% – 250% = 705%

4 After 9 years population $= 800\,000 \times (1.15)^9$
$= 2\,814\,301$
After 10 years population $= 800\,000 \times (1.15)^{10}$
$= 3\,236\,446$
So population would exceed 3 000 000 for the first time in the tenth year, i.e. in 2007

5 **a** Simple interest $= 5\,000\,000 \times 0.05 \times 12$
$= €3\,000\,000$

b Compound interest
$= 5\,000\,000 \times (1.05)^{12} - 5\,000\,000$
$= €3\,979\,282$

6 After 5 years value is $(1.125)^5 = 1.80$ times its initial value
After 6 years value is $(1.125)^6 = 2.03$ times its initial value – so it will take 6 years to double in value.

7 After 10 days the population is $(1.07)^{10}$
$= 1.967$ times its initial value
After 11 days the population is $(1.07)^{11} = 2.10$ times its initial value – so it will take 11 days for the population to double.

8 **a** Simple interest $= 4\,000\,000 \times 0.085 \times 3$
$= \$1\,020\,000$

b Compound interest
$= 4\,000\,000 \times (1.085)^3 - 4\,000\,000$
$= \$1\,109\,000$

c Quarterly rate $= \frac{8.5}{4} = 2.125\%$
Compound interest calculated quarterly
$= 4\,000\,000 \times (1.02125)^{12} - 4\,000\,000$
$= \$1\,148\,000$

9 After 10 years value is $(0.88)^{10} = 0.279$ times its initial value
After 11 years value is $(0.88)^{11} = 0.245$ times its initial value – so it will take 11 years before the car is only worth 25% of its original value.

Exercise 3.7.6

4 **a** Yes

b **i** $P(A \cup B)' = 0.1$

ii $P(A \cup B) = 1 - P(A \cup B)' = 1 - 0.1 = 0.9$

iii $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.9 = 0.7 + 0.5 - P(A \cap B)$
$(A \cap B) = 1.2 - 0.9 = 0.3$

iv $P(A \setminus B) = \frac{P(A \cap B)}{P(B)} = 0.\frac{3}{0}.5 = 0.6$

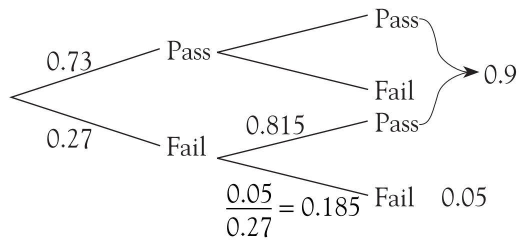
5 **a** $P(B \cap L) = 0.75 \times 0.9 = 0.675$

b $P(B' \cap L) = 0.25 \times 0.8 = 0.2$

c $P(L) = P(B \cap L) + P(B' \cap L)$
$= 0.675 + 0.2$
$= 0.875$

d $P(B \setminus L) = \frac{P(B \cap L)}{P(L)}$
$= \frac{0.675}{0.875}$
$= 0.77$ (2 d.p.)

6 Driving test (X) Drama exam (Y)



Given information:
$P(X) = 0.73$
$P(Y) = 0.9 = P(X \cap Y) + P(X' \cap Y)$
$P(X' \cap Y') = 0.05$
We are required to calculate $P(Y \setminus X)$

$P(Y \setminus X) = \frac{P(Y \cap X)}{P(X)}$
$P(Y \cap X) = 0.9 - P(X' \cap Y)$
$= 0.9 - (0.27 \times 0.815)$
$= 0.68$

So, $P(Y \setminus X) = \frac{0.68}{0.73}$
$= 0.93$

7 Let 1 = Gold in 100 m freestyle
Let 2 = Gold in 200 m freestyle
Information given:
$P(1) = 0.6$
$P(2) = 0.7\ (= P(1 \cap 2) + P(1' \cap 2))$
$P(1' \cap 2') = 0.1$
We need $P(2 \setminus 1) = \frac{P(1 \cap 2)}{P(1)}$
$P(1 \cap 2) = 0.7 - (0.4 \times 0.75) = 0.4$
So $P(2 \setminus 1) = \frac{0.4}{0.6} = 0.66$

Exercise 5.1.4

1 a Gradient $= \frac{7-1}{4-1} = \frac{6}{3} = 2$

So equation is $y = 2x + c$

Substituting (1, 1): $1 = 2 + c$, i.e. $c = -1$

So i) $y = 2x - 1$ or ii) $2x - y - 1 = 0$

b Gradient $= \frac{10-4}{3-1} = \frac{6}{2} = 3$

So equation is $y = 3x + c$

Substituting (1, 4): $4 = 3 + c$, i.e. $c = 1$

So **i)** $y = 3x + 1$ or **ii)** $3x - y + 1 = 0$

c Gradient $= \frac{7-5}{2-1} = 2$

So equation is $y = 2x + c$

Substituting (1, 5): $5 = 2 + c$, i.e. $c = 3$

So **i)** $y = 2x + 3$ or **ii)** $2x - y + 3 = 0$

d Gradient $= \frac{-1 - -4}{3-0} = \frac{3}{3} = 1$

So equation is $y = x + c$

Substituting (0, –4): $-4 = c$

So **i)** $y = x - 4$ or **ii)** $x - y - 4 = 0$

e Gradient $= \frac{10-6}{2-1} = 4$

So equation is $y = 4x + c$

Substituting (1, 6): $6 = 4 + c$, i.e. $c = 2$

So **i)** $y = 4x + 2$ or **ii)** $4x - y + 2 = 0$

f Gradient $= \frac{4-3}{0-1} = -1$

So equation is $y = -x + c$

Substituting (0, 4): $4 = c$

So **i)** $y = -x + 4$ or **ii)** $x + y - 4 = 0$

g Gradient $= \frac{-4 - -18}{3-10} = \frac{14}{-7} = -2$

So equation is $y = -2x + c$

Substituting (3, –4): $-4 = -6 + c$, i.e. $c = 2$

So **i)** $y = -2x + 2$ or **ii)** $2x + y - 2 = 0$

h Gradient $= \frac{-1 - -4}{0-1} = \frac{3}{-1} = -3$

So equation is $y = -3x + c$

Substituting (0, –1): $-1 = 0 + c$, i.e. $c = -1$

So **i)** $y = -3x - 1$ or **ii)** $3x + y + 1 = 0$

i Gradient $= \frac{5-0}{10-0} = \frac{1}{2}$

So equation is $y = \frac{1}{2}x + c$

Substituting (0, 0): $0 = c$

So **i)** $y = \frac{1}{2}x$ or **ii)** $x - 2y = 0$

2 a Gradient $= \frac{4-3}{2 - -5} = \frac{1}{7}$

So equation is $y = \frac{1}{7}x + c$

Substituting (2, 4): $4 = \frac{2}{7} + c$, i.e. $c = \frac{26}{7}$

So **i)** $y = \frac{1}{7}x + \frac{26}{7}$ or **ii)** $x - 7y + 26 = 0$

b Gradient $= \frac{4 - -2}{4 - -3} = \frac{6}{7}$

So equation is $y = \frac{6}{7}x + c$

Substituting (4, 4): $4 = \frac{24}{7} + c$, i.e. $c = \frac{4}{7}$

So **i)** $y = \frac{6}{7}x + \frac{4}{7}$ or **ii)** $6x - 7y + 4 = 0$

c Gradient $= \frac{6 - -3}{-1 - -7} = \frac{9}{6} = \frac{3}{2}$

So equation is $y = \frac{3}{2}x + c$

Substituting (–1, 6): $6 = -\frac{3}{2} + c$, i.e. $c = \frac{15}{2}$

So **i)** $y = \frac{3}{2}x + \frac{15}{2}$ or **ii)** $3x - 2y + 15 = 0$

d Gradient $= \frac{5 - -4}{2-1} = 9$

So equation is $y = 9x + c$

Substituting (2, 5): $5 = 18 + c$, i.e. $c = -13$

So **i)** $y = 9x - 13$ or **ii)** $9x - y - 13 = 0$

e Gradient $= \frac{4-0}{-3-5} = -\frac{4}{8} = -\frac{1}{2}$

So equation is $y = -\frac{1}{2}x + c$

Substituting (5, 0): $0 = -\frac{5}{2} + c$, i.e., $c = \frac{5}{2}$

So **i)** $y = -\frac{1}{2}x + \frac{5}{2}$ or **ii)** $x + 2y - 5 = 0$

f Gradient $= \frac{4-7}{6 - -7} = -\frac{3}{13}$

So equation is $y = -\frac{3}{13}x + c$

Substituting (6, 4): $4 = -\frac{18}{13} + c$, i.e., $c = \frac{70}{13}$

So **i)** $y = -\frac{3}{13}x + \frac{70}{13}$ or **ii)** $3x + 13y - 70 = 0$

g Gradient $= \frac{2-2}{6-5} = 0$

So equation is $y = c$ with $c = 2$

So **i)** $y = 2$ or **ii)** $y - 2 = 0$

h Gradient $= \frac{6 - -3}{-2-1} = -\frac{9}{3} = -3$

So equation is $y = -3x + c$

Substituting (1, –3): $-3 = -3 + c$, i.e., $c = 0$

So **i)** $y = -3x$ or **ii)** $3x + y = 0$

i Gradient $= \frac{6 - -4}{6-6} = \frac{10}{0}$, i.e. a vertical line

So equation is $x = c$ with $c = 6$

So **i)** $x = 6$ or **ii)** $x - 6 = 0$

Exercise 5.1.6

1 a $x + y = 6$ (1)
$x - y = 2$ (2)
(1) + (2): $2x = 8$, i.e., $x = 4$
In (1): $4 + y = 6$, so $y = 2$

b $x + y = 11$ (1)
$x - y - 1 = 0$ (2)
(1) + (2): $2x - 1 = 11$, i.e., $2x = 12$, $x = 6$
In (1): $6 + y = 11$, so $y = 5$

c $x + y = 5$ (1)
$x - y = 7$ (2)
(1) + (2): $2x = 12$, i.e., $x = 6$
In (1): $6 + y = 5$, so $y = -1$

d $2x + y = 12$ (1)
$2x - y = 8$ (2)
(1) + (2): $4x = 20$, i.e., $x = 5$
In (1): $10 + y = 12$, so $y = 2$

e $3x + y = 17$ (1)
$3x - y = 13$ (2)
(1) + (2): $6x = 30$, i.e., $x = 5$
In (1): $15 + y = 17$, so $y = 2$

f $5x + y = 29$ (1)
$5x - y = 11$ (2)
(1) + (2): $10x = 40$, i.e., $x = 4$
In (1): $20 + y = 29$, so $y = 9$

2 a $3x + 2y = 13$ (1)
$4x = 2y + 8$ (2)
Rearrange (2): $4x - 2y = 8$
(1) + (2): $7x = 21$, i.e., $x = 3$
In (1): $9 + 2y = 13$, $2y = 4$, so $y = 2$

b $6x + 5y = 62$ (1)
$4x - 5y = 8$ (2)
(1) + (2): $10x = 70$, i.e., $x = 7$
In (2): $28 - 5y = 8$, $5y = 20$, so $y = 4$

c $x + 2y = 3$ (1)
$8x - 2y = 6$ (2)
(1) + (2): $9x = 9$, i.e., $x = 1$
In (1): $1 + 2y = 3$, $2y = 2$, so $y = 1$

d $9x + 3y = 24$ (1)
$x - 3y = -14$ (2)
(1) + (2): $10x = 10$, i.e., $x = 1$
In (1): $9 + 3y = 24$, $3y = 15$, so $y = 5$

e $7x - y = -3$ (1)
$4x + y = 14$ (2)
(1) + (2): $11x = 11$, i.e., $x = 1$
In (2): $4 + y = 14$, so $y = 10$

f $3x = 5y + 14$ (1)
$6x + 5y = 58$ (2)
(1) + (2): $9x + 5y = 5y + 72$, i.e., $9x = 72$, so $x = 8$
In (1): $24 = 5y + 14$, $5y = 10$, so $y = 2$

3 a $2x + y = 14$ (1)
$x + y = 9$ (2)
(2): × 2: $2x + 2y = 18$ (3)
(3) – (1): $y = 4$
In (1): $2x + 4 = 14$, $2x = 10$, so $x = 5$

b $5x + 3y = 29$ (1)
$x + 3y = 13$ (2)
(1) – (2): $4x = 16$, i.e., $x = 4$
In (2): $4 + 3y = 13$, $3y = 9$, so $y = 3$

c $4x + 2y = 50$ (1)
$x + 2y = 20$ (2)
(1) – (2): $3x = 30$, i.e., $x = 10$
In (2): $10 + 2y = 20$, $2y = 10$, so $y = 5$

d $x + y = 10$ (1)
$3x = -y + 22$ (2)
Rearrange (2): $3x + y = 22$ (3)
(3) – (1): $2x = 12$, i.e., $x = 6$
In (1): $6 + y = 10$, so $y = 4$

e $2x + 5y = 28$ (1)
$4x + 5y = 36$ (2)
(2) – (1): $2x = 8$, i.e., $x = 4$
In (1): $8 + 5y = 28$, $5y = 20$, so $y = 4$

f $x + 6y = -2$ (1)
$3x + 6y = 18$ (2)
(2) – (1): $2x = 18 - -2$, $2x = 20$, i.e., $x = 10$
In (1): $10 + 6y = -2$, $6y = -12$, so $y = -2$

4 a $x - y = 1$ (1)
$2x - y = 6$ (2)
(2) – (1): $x = 5$
In (1): $5 - y = 1$, so $y = 4$

b $3x - 2y = 8$ (1)
$2x - 2y = 4$ (2)
(1) – (2): $x = 4$
In (1): $12 - 2y = 8$, $2y = 4$, so $y = 2$

c $7x - 3y = 26$ (1)
$2x - 3y = 1$ (2)
(1) – (2): $5x = 25$, i.e., $x = 5$
In (2): $10 - 3y = 1$, $3y = 9$, so $y = 3$

d $x = y + 7$ (1)
$3x - y = 17$ (2)
Rearrange (2): $3x = y + 17$ (3)
(3) – (1): $2x = 10$, i.e., $x = 5$
In (1): $5 = y + 7$, so $y = -2$

e $8x - 2y = -2$ (1)
$3x - 2y = -7$ (2)
(1) – (2): $5x = -2 + 7$, $5x = 5$, i.e., $x = 1$
In (1): $8 - 2y = -2$, $2y = 10$, so $y = 5$

f $4x - y = -9$ (1)
$7x - y = -18$ (2)
(2) – (1): $3x = -18 + 9$, $3x = -9$, i.e., $x = -3$
In (1): $-12 - y = -9$, so $y = -3$

5 a $x + y = -7$ (1)
$x - y = -3$ (2)
(1) + (2): $2x = -10$, i.e., $x = -5$
In (1): $-5 + y = -7$, so $y = -2$

b $2x + 3y = -18$ (1)
$2x = 3y + 6$ (2)
Rearrange (2): $2x - 3y = 6$ (3)
(1) + (3): $4x = -12$, i.e., $x = -3$
In (2): $-6 = 3y + 6$, $3y = -12$, so $y = -4$

c $5x - 3y = 9$ (1)
$2x + 3y = 19$ (2)
(1) + (2): $7x = 28$, i.e., $x = 4$
In (2): $8 + 3y = 19$, $3y = 11$, so $y = \frac{11}{3} = 3\frac{2}{3}$

d $7x + 4y = 42$ (1)
$9x - 4y = -10$ (2)
(1) + (2): $16x = 32$, i.e., $x = 2$
In (1): $14 + 4y = 42$, $4y = 28$, so $y = 7$

e $4x - 4y = 0$ (1)
$8x + 4y = 12$ (2)
(1) + (2): $12x = 12$, i.e., $x = 1$
In (1): $4 - 4y = 0$, so $y = 1$

f $x - 3y = -25$ (1)
$5x - 3y = -17$ (2)
(2) – (1): $4x = -17 - -25$, $4x = 8$, i.e., $x = 2$
In (1): $2 - 3y = -25$, $3y = 27$, so $y = 9$

6 a $2x + 3y = 13$ (1)
$2x - 4y + 8 = 0$ (2)
Rearrange (2): $2x - 4y = -8$ (3)
(1) – (3): $3y - -4y = 13 - -8$, $7y = 21$, i.e., $y = 3$
In (1): $2x + 9 = 13$, $2x = 4$ so $x = 2$

b $2x + 4y = 50$ (1)
$2x + y = 20$ (2)
(1) – (2): $3y = 30$, i.e., $y = 10$
In (2): $2x + 10 = 20$, $2x = 10$, so $x = 5$

c $x + y = 10$ (1)
$3y = 22 - x$ (2)
Rearrange (1): $x = 10 - y$
Substitute into (2): $3y = 22 - (10 - y)$
$2y = 12$, i.e. $y = 6$
In (1): $x + 6 = 10$, so $x = 4$

d $5x + 2y = 28$ (1)
$5x + 4y = 36$ (2)
(2) – (1): $2y = 8$, i.e., $y = 4$
In (1): $5x + 8 = 28$, $5x = 20$, so $x = 4$

7 a $-4x = 4y$ (1)
$4x - 8y = 12$ (2)
From (1) $y = -x$
In (2): $4x + 8x = 12$, i.e., $x = 1$ and $y = -1$

b $3x = 19 + 2y$ (1)
$-3x + 5y = 5$ (2)
(1) + (2): $0 + 5y = 24 + 2y$, i.e., $3y = 24$, so $y = 8$
In (1): $3x = 19 + 16$, $3x = 35$, so $x = \frac{35}{3} = 11\frac{2}{3}$

c $3x + 2y = 12$ (1)
$-3x + 9y = -12$ (2)
(1) + (2): $11y = 0$, i.e., $y = 0$
In (1): $3x = 12$, so $x = 4$

d $3x + 5y = 29$ (1)
$3x + y = 13$ (2)
(1) – (2): $4y = 16$, i.e., $y = 4$
In (2): $3x + 4 = 13$, $3x = 9$, so $x = 3$

e $-5x + 3y = 14$ (1)
$5x + 6y = 58$ (2)
(1) + (2): $9y = 72$, i.e., $y = 8$
In (2): $5x + 48 = 58$, i.e., $5x = 10$, so $x = 2$

f $-2x + 8y = 6$ (1)
$2x = 3 - y$ (2)
(1) + (2): $0 + 8y = 9 - y$, i.e., $9y = 9$, so $y = 1$
In (2): $2x = 3 - 1$, $2x = 2$, so $x = 1$

Exercise 5.1.7

1 a $2x + y = 7$ (1)
$3x + 2y = 12$ (2)
(1) × 2: $4x + 2y = 14$ (3)
(3) – (2): $x = 2$
In (1): $4 + y = 7$, so $y = 3$

b $5x + 4y = 21$ (1)
$x + 2y = 9$ (2)
(2) × 2: $2x + 4y = 18$ (3)
(1) – (3): $3x = 3$ i.e., $x = 1$
In (2): $1 + 2y = 9$, $2y = 8$, so $y = 4$

c $x + y = 7$ (1)
$3x + 4y = 23$ (2)
(1) × 3: $3x + 3y = 21$ (3)
(2) – (3): $y = 2$
In (1): $x + 2 = 7$, so $x = 5$

d $2x - 3y = -3$ (1)
$3x + 2y = 15$ (2)
$(1) \times 2$: $4x - 6y = -6$ (3)
$(2) \times 3$: $9x + 6y = 45$ (4)
$(3) + (4)$: $13x = 39$ i.e., $x = 3$
In (1): $6 - 3y = -3$, $9 = 3y$, so $y = 3$

e $4x = 4y + 8$ (1)
$x + 3y = 10$ (2)
$(2) \times 4$: $4x + 12y = 40$
$4x = -12y + 40$ (3)
$(1) - (3)$: $0 = 16y - 32$, $16y = 32$, i.e., $y = 2$
In (1): $4x = 8 + 8 = 16$, so $x = 4$

f $x + 5y = 11$ (1)
$2x - 2y = 10$ (2)
$(1) \times 2$: $2x + 10y = 22$ (3)
$(3) - (2)$: $12y = 12$, i.e., $y = 1$
In (1): $x + 5 = 11$, so $x = 6$

2 a $x + y = 5$ (1)
$3x - 2y = -5$ (2)
$(1) \times 2$: $2x + 2y = 10$ (3)
$(1) + (2)$: $5x = 5$, i.e., $x = 1$
In (1): $1 + y = 5$, so $y = 4$

b $2x - 2y = 6$, i.e., $x - y = 3$ (1)
$x - 5y = -5$ (2)
$(1) - (2)$: $4y = 8$, i.e., $y = 2$
In (1): $2x - 4 = 6$, so $x = 5$

c $2x + 3y = 15$ (1)
$3x + 2y = 15$ (2)
$(1) \times 3$: $6x + 9y = 45$ (3)
$(2) \times 2$: $6x + 4y = 30$ (4)
$(3) - (4)$: $5y = 15$, i.e., $y = 3$
In (1): $2x + 9 = 15$, so $x = 3$

d $x - 6y = 0$ (1)
$3x - 3y = 15$, i.e., $x - y = 5$ (2)
$(2) - (1)$: $5y = 5$, i.e., $y = 1$
In (1): $x - 6 = 0$, so $x = 6$

e $2x - 5y = -11$ (1)
$3x + 4y = 18$ (2)
$(1) \times 3$: $6x - 15y = -33$ (3)
$(2) \times 2$: $6x + 8y = 36$ (4)
$(4) - (3)$: $23y = 69$, i.e., $y = 3$
In (2): $3x + 12 = 18$, $3x = 6$, so $x = 2$

f $x + y = 5$ (1)
$2x - 2y = -2$, i.e., $x - y = -1$ (2)
$(1) + (2)$: $2x = 4$, i.e., $x = 2$
In (1): $2 + y = 5$, so $y = 3$

3 a $-2x + 3y = 9$ (1)
$3x + 2y = 6$ (2)
$(1) \times 3$: $-6x + 9y = 27$ (3)
$(2) \times 2$: $6x + 4y = 12$ (4)
$(3) + (4)$: $13y = 39$, i.e., $y = 3$
In (2): $3x + 6 = 6$, so $x = 0$

b $x + 4y = 13$ (1)
$3x - 3y = 9$ (2)
$(1) \times 3$: $3x + 12y = 39$ (3)
$(3) - (2)$: $15y = 30$, i.e., $y = 2$
In (1): $x + 8 = 13$, so $x = 5$

c $2x - 3y = -19$ (1)
$3x + 2y = 17$ (2)
$(1) \times 2$: $4x - 6y = -38$ (3)
$(2) \times 3$: $9x + 6y = 51$ (4)
$(3) + (4)$: $13x = 13$, i.e., $x = 1$
In (2): $3 + 2y = 17$, $2y = 14$, so $y = 7$

d $2x - 5y = -8$ (1)
$-3x - 2y = -26$ (2)
$(1) \times 3$: $6x - 15y = -24$ (3)
$(2) \times 2$: $-6x - 4y = -52$ (4)
$(3) + (4)$: $-19y = -76$, i.e., $y = 4$
In (1): $2x - 20 = -8$, $2x = 12$, so $x = 6$

e $5x - 2y = 0$ (1)
$2x + 5y = 29$ (2)
$(1) \times 5$: $25x - 10y = 0$ (3)
$(2) \times 2$: $4x + 10y = 58$ (4)
$(3) + (4)$: $29x = 58$, i.e., $x = 2$
In (1): $10 - 2y = 0$, $2y = 10$, so $y = 5$

f $x + 8y = 3$ (1)
$3x - 2y = 9$ (2)
$(2) \times 4$: $12x - 8y = 36$ (3)
$(3) + (1)$: $13x = 39$, i.e., $x = 3$
In (1): $3 + 8y = 3$, so $y = 0$

4 a $4x + 2y = 5$ (1)
$3x + 6y = 6$ (2)
$(1) \times 3$: $12x + 6y = 15$ (3)
$(3) - (2)$: $9x = 9$, i.e., $x = 1$
In (1): $4 + 2y = 5$, so $y = 0.5$

b $4x + y = 14$ (1)
$6x - 3y = 3$ (2)
$(1) \times 3$: $12x + 3y = 42$ (3)
$(2) + (3)$: $18x = 45$, $2x = 5$, i.e., $x = 2.5$
In (1): $10 + y = 14$, so $y = 4$

c $10x - y = -2$ (1)
$-15x + 3y = 9$ (2)
$(1) \times 3$: $30x - 3y = -6$ (3)
$(2) + (3)$: $15x = 3$, i.e., $x = \frac{1}{5}$
In (1): $2 - y = -2$, so $y = 4$

d $2x - 2y = 0.5$ (1)
$6x + 3y = 6$ (2)
$(1) \times 3$: $6x - 6y = 1.5$ (3)
$(2) - (3)$: $9y = 4.5$, i.e., $y = \frac{1}{2}$
In (1): $2x - 1 = 0.5$, $2x = 1.5$, so $x = \frac{3}{4}$

e $x + 3y = 6$ (1)
$2x - 9y = 7$ (2)
$(1) \times 2$: $2x + 6y = 12$ (3)
$(3) - (2)$: $15y = 5$, i.e., $y = \frac{1}{3}$
In (1): $x + 1 = 6$, so $x = 5$

f $5x - 3y = -0.5$ (1)
$3x + 2y = 3.5$ (2)
$(1) \times 2$: $10x - 6y = -1$ (3)
$(2) \times 3$: $9x + 6y = 10.5$ (4)
$(3) + (4)$: $19x = 9.5$, i.e., $x = \frac{1}{2}$
In (2): $1.5 + 2y = 3.5$, so $y = 1$

Exercise 5.2.1

1 a $\tan 58° = \frac{x}{12}$, $x = 12 \tan 58° = 19.2$ cm

b $\cos 15° = \frac{14.6}{y}$, $y = \frac{14.6}{\cos 15°} = 15.1$ cm

c $\sin 22° = \frac{16.4}{k}$, $k = \frac{16.4}{\sin 22°} = 43.8$ cm

d $\sin 45° = \frac{c}{45}$, $c = 45 \sin 45° = 31.8$ cm

e $\tan 56° = \frac{9.2}{x}$, $x = \frac{9.2}{\tan 56°} = 6.2$ cm

f $\cos 81° = \frac{a}{13.7}$, $a = 13.7 \cos 81° = 2.1$ cm

2 a $\cos x = \frac{8.1}{52.3}$
$x = \cos^{-1}\left(\frac{8.1}{52.3}\right) = 81.1°$

b $\tan x = \frac{8}{4}$
$x = \tan^{-1}(2) = 63.4°$

c $\sin x = \frac{5}{8}$
$x = \sin^{-1}\left(\frac{5}{8}\right) = 38.7°$

3 a $\sin x = \frac{6}{8.7}$
$x = \sin^{-1}\left(\frac{6}{8.7}\right) = 43.6°$

b $\tan 38° = \frac{15.2}{l}$, $l = \frac{15.2}{\tan 38°} = 19.5$ cm

c $\cos a = \frac{14}{19}$
$a = \cos^{-1}\left(\frac{14}{19}\right) = 42.5°$

4 a $ZX^2 = XY^2 + YZ^2$
$ZX = \sqrt{12^2 + 17^2} = 20.8$ km

b $12 \tan YZX = 17$
$YZX = \tan^{-1}\left(\frac{17}{12}\right) = 54.8$
Bearing is $180 + (90 - 54.8) = 215.2°$

5 a $250 \cos 24° = 228$ km
b $250 \sin 24° = 102$ km
c $180 \cos 55° = 103$ km
d $180 \sin 55° = 147$ km
e $GJ^2 = (103 + 228)^2 + (102 + 147)^2$
$GJ = 415$ km
f $(103 + 228) \tan x = 102 + 147$
$x = \tan^{-1}\left(\frac{249}{331}\right) = 37°$
Bearing is $180 + 37 = 217°$

6 a Height of small tree $= 8 \tan 40° = 6.7$ m
b Height of tall tree $= 6.7 + 20 \sin 40° = 19.6$ m
c Horizontal distance between trees $=$
$20 \cos 40° = 15.3$ m

7 a $9 \cos SQR = 6$
angle $SQR = \cos^{-1}\left(\frac{6}{9}\right) = 48.2°$

b angle PSR = 90° (SR parallel to PQ),
so angle PSQ $= 90 - 48.2 = 41.8°$
c $PQ = 12 \sin 41.8° = 8.0$ cm
d $PS = 12 \cos 41.8° = 8.9$ cm
e Area PQRS = area PQS + area SQR
$= \left(\frac{1}{2} \times 12 \times 8 \sin 48.2°\right) + \left(\frac{1}{2} \times 12 \times 9 \sin 48.2°\right)$
$= 76.0 \text{ cm}^2$

8 a Vertical height $= 1 \times \sin 20° = 0.342$ km
$= 342$ m
b Horizontal distance $= 1 \times \cos 20° = 0.940$ km
$= 940$ m

9 $AB = 6 \tan 60° - 6 \tan 30° = 6.9$ km

10 a Height $= 130 \tan 60° = 225$ m
If x is the angle at B, $130 \tan 60° = 200 \tan x$
So $x = \tan^{-1}\left(\frac{130 \tan 60}{200}\right) = 48.4°$

Exercise 5.3.3

1 Using the sine rule

a $\dfrac{x}{\sin 40°} = \dfrac{12}{\sin 60°}$

$x = \dfrac{12\sin 40°}{\sin 60°} = 8.9$ cm

b $\dfrac{x}{\sin 20°} = \dfrac{20}{\sin 130°}$

$x = \dfrac{20\sin 20°}{\sin 130°} = 8.9$ cm

c $\dfrac{x}{\sin 35°} = \dfrac{9}{\sin(180 - 25 - 35)°}$

$x = \dfrac{9\sin 35°}{\sin 120°} = 6.0$ mm

d In the isosceles triangle equal angles are 40°. So unmarked acute angle in small triangle is (80 – 40) = 40°. Let y be the side common to both triangles. By the sine rule,

$\dfrac{y}{\sin 110°} = \dfrac{3}{\sin 30°}$

$y = \dfrac{3\sin 110°}{\sin 30°}$

In the large triangle,

$\dfrac{y}{\sin 40°} = \dfrac{x}{\sin 100}$

$x = \dfrac{3\sin 110°\sin 100°}{\sin 30°\sin 40°} = 8.6$ cm

2 a $\dfrac{\sin\theta}{8} = \dfrac{\sin 20°}{5}$

$\theta = \sin^{-1}\dfrac{8\sin 20°}{5} = 33.2°$

b $\dfrac{\sin\theta}{35} = \dfrac{\sin 30°}{22}$

$\theta = \sin^{-1}\dfrac{35\sin 30°}{22} = 52.7°$ or $(180 - 52.7)°$

θ is obtuse = 127.3°

c $\dfrac{\sin\theta}{9} = \dfrac{\sin 60°}{8}$

$\theta = \sin^{-1}\dfrac{9\sin 60°}{8} = 77.0°$

d Let side opposite to θ be x. Then

$\dfrac{x}{\sin 50°} = \dfrac{7}{\sin 40°}$

By the sine rule again,

$\dfrac{\sin\theta}{x} = \dfrac{\sin 30°}{6}$

$\sin\theta = \dfrac{7\sin 50°\sin 30°}{6\sin 40°}$

$\theta = \sin^{-1}\left(\dfrac{7\sin 50°\sin 30°}{6\sin 40°}\right) = 44.0°$

3 a $\dfrac{\sin ABC}{10} = \dfrac{\sin 20°}{8}$

sin ABC = 0.4275

ABC can be 25.3° or (180 – 25.3) = 154.7°

4 a $\dfrac{\sin QRP}{6} = \dfrac{\sin 40°}{4}$

$QRP = \sin^{-1}\left(\dfrac{6\sin 40°}{4}\right) = 74.6°$

and (180 – 74.6) = 105.4°

Exercise 5.3.4

1 Using the cosine rule,

a $x^2 = 3^2 + 2^2 - (2 \times 3 \times 2 \times \cos 140°)$

$x = 4.7$ m

b $x^2 = 10^2 + 6^2 - (2 \times 10 \times 6 \times \cos 95°)$

$x = 12.1$ cm

c $x^2 = 15^2 + 7^2 - (2 \times 7 \times 15 \times \cos 25°)$

$x = 9.1$ m

d $x^2 = 4^2 + 4^2 - (2 \times 4 \times 4 \times \cos 45°)$

$x = 3.1$ cm

e $x^2 = 5^2 + 7^2 - (2 \times 7 \times 5 \times \cos 125°)$

$x = 10.7$ m

2 Using the cosine rule,

a $40^2 = 20^2 + 25^2 - (2 \times 20 \times 25 \times \cos\theta)$

$\theta = \cos^{-1}\left(\dfrac{20^2 + 25^2 - 40^2}{2 \times 20 \times 25}\right) = 125.1°$

b $5^2 = 2^2 + 4^2 - (2 \times 4 \times 2 \times \cos\theta)$

$\theta = \cos^{-1}\left(\dfrac{2^2 + 4^2 - 5^2}{2 \times 2 \times 4}\right) = 108.2°$

c Let the obtuse angle be

$15^2 = 9^2 + 9^2 - (2 \times 9 \times 9 \times \cos\phi)$

$\theta = \cos^{-1}\left(\dfrac{9^2 + 9^2 - 15^2}{2 \times 9 \times 9}\right) = 112.9°$

As triangle is isosceles,

$= \frac{1}{2}(180 - 112.9) = 33.6°$

d $15^2 = 4^2 + 18^2 - (2 \times 4 \times 18 \times \cos\theta)$

$\theta = \cos^{-1}\left(\dfrac{4^2 + 18^2 - 15^2}{2 \times 4 \times 18}\right) = 37.0°$

e $15^2 = 10^2 + 7^2 - (2 \times 10 \times 7 \times \cos\theta)$

$\theta = \cos^{-1}\left(\frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}\right) = 122.9°$

3 a $XZ^2 = 40^2 + 24^2 - (2 \times 40 \times 24 \times \cos 80°)$

$XZ = 42.9$

b $42.9^2 = 20^2 + 30^2 - (2 \times 20 \times 30 \times \cos ZWX)$

$\text{Angle ZWX} = \cos^{-1}\left(\frac{20^2 + 30^2 - 42.9^2}{2 \times 20 \times 30}\right)$

$= 116.9°$

c Using the sine rule,

$\frac{\sin WZX}{20} = \frac{\sin 116.9°}{42.9}$

$\text{Angle WZX} = \sin^{-1}\left(\frac{20 \sin 116.9°}{42.9}\right) = 24.6°$

d Using the sine rule,

$\frac{\sin YZX}{24} = \frac{\sin 80°}{42.9}$

$\text{Angle YZX} = \sin^{-1}\left(\frac{24 \sin 80°}{42.9}\right) = 33.4°$

e Angle WZY = angle WZX + YZX

$= 24.6 + 33.4 = 58°$

Using the cosine rule,

$WY^2 = 30^2 + 40^2 - (2 \times 30 \times 40 \times \cos 58°)$

WY = 35.0 m

4 $BC^2 = 220^2 + 450^2 - (2 \times 220 \times 450 \times \cos 55°)$

BC = 370 m (to nearest 10 m)

5 a Equating expressions for the height:

$AX \sin 60° = AY \sin 45°$ (1)

We also know that

$AX \cos 60° + 200 = AY \cos 45°$ (2)

Substituting for AX in (2):

$\frac{AY \sin 45° \cos 60°}{\sin 60°} + 200 = AY \cos 45°$

$AY\left(\cos 45° - \frac{\sin 45° \cos 60°}{\sin 60°}\right) = 200$

AY = 669 m

b Substituting back into (1) in part **a**

$AX = 669 \times \frac{\sin 45°}{\sin 60°} = 546\,\text{m}$

c Height of cliff = AY sin 45° = 473 m

6 Using the cosine rule,

$\text{Distance}^2 = 40^2 + 50^2 - (2 \times 40 \times 50 \times \cos 110°)$

Distance = 73.9 m

Exercise 5.3.5

1 a $\text{Area} = 0.5 \times 20 \times 14 \times \sin 30° = 70.0\,\text{cm}^2$

b $\text{Area} = 0.5 \times 12 \times 12 \times \sin 80° = 70.9\,\text{mm}^2$

c $\text{Area} = 0.5 \times 40 \times 35 \times \sin 10° = 121.6\,\text{cm}^2$

d $\text{Area} = 0.5 \times 8 \times 6 \times \sin 135° = 17.0\,\text{cm}^2$

2 a $40 = 0.5 \times 12 \times 16 \times \sin x$

$x = \sin^{-1}\left(\frac{5}{12}\right) = 24.6°$

b $20 = 0.5 \times 9 \times x \times \sin 160°$

$x = \frac{20 \times 2}{9 \sin 160°} = 13.0\,\text{cm}$

c $150 = 0.5 \times 15 \times x \times \sin 60°$

$x = \frac{20}{\sin 60°} = 23.1\,\text{cm}$

d $50 = 0.5 \times 14 \times 8 \times \sin x$

$x = \sin^{-1}\left(\frac{50}{7 \times 8}\right) = 63.2°$

3 Area = area OAB + area OBC + area OCD + area ODA

$= (0.5 \times 83 \times 122 \times \sin 100°) +$
$(0.5 \times 122 \times 106 \times \sin 60°) +$
$(0.5 \times 106 \times 78 \times \sin 130°) +$
$(0.5 \times 78 \times 83 \times \sin 70°)$

$= 16\,972$

$= 16\,800\,\text{m}^2$ (to nearest $100\,\text{m}^2$)

4 a CSA of roof $= 0.5 \times 3 \times 3 \times \sin 120° = 3.9\,\text{m}^2$

b Volume of garage

$= (3.9 \times 9) + (2 \times 3 \times \sin 60° \times 4 \times 9)$

$= 222\,\text{m}^2$

Exercise 5.4.1

1 a $HF^2 = HG^2 + FG^2 = 4^2 + 4^2$

$HF = \sqrt{32} = 5.7\,\text{cm}$

b $HB^2 = HF^2 + BF^2 = 32 + 4^2 = 48$

$HB = \sqrt{48} = 6.9\,\text{cm}$

c Angle between HF and HB

$\sqrt{48} \cos BHG = 4$

$BHG = \cos^{-1}\left(\frac{4}{\sqrt{48}}\right) = 54.7$

d $XY^2 = 2^2 + 2^2 = 8$

$XY = \sqrt{8} = 2.8\,\text{cm}$

2 a $CA^2 = 3^2 + 5^2 = 34$

$CA = \sqrt{34} = 5.8\,\text{cm}$

b $CE^2 = EG^2 + CG^2$
Now length EG = length CA
$CE^2 = 34 + 2^2 = 38$
$CE = \sqrt{38} = 6.2\,\text{cm}$

c $\sqrt{34}\tan ACE = 2$
$\text{Angle } ACE = \tan^{-1}\left(\frac{2}{\sqrt{34}}\right) = 18.9°$

3 a $EG^2 = EH^2 + HG^2 = 5^2 + 4^2 = 41$
$EG = \sqrt{41} = 6.4\,\text{cm}$

b $AG^2 = EG^2 + AE^2 = 41 + 12^2 = 185$
$AG = \sqrt{185} = 13.6\,\text{cm}$

c $\text{Angle } AGE = \tan^{-1}\left(\frac{12}{\sqrt{41}}\right) = 61.9°$

4 a $EB^2 = 7^2 + 3^2 = 58$
$EB = \sqrt{58}$
$2\tan\theta = \sqrt{58}$
$\theta = \tan^{-1}\left(\frac{\sqrt{58}}{2}\right) = 75.3°$

b $HF^2 = 3^2 + 2^2 = 13$
$HF = \sqrt{13}$
$\sqrt{13}\cos\theta = 2$
$\cos^{-1}\left(\frac{2}{\sqrt{13}}\right) = 56.3°$

5 a i $DB^2 = 6^2 + 4^2 = 36 + 16 = 52$
$DB = \sqrt{52} = 7.2\,\text{cm}$

ii $DX = \frac{\sqrt{52}}{2}$
$10\sin DAX = \frac{\sqrt{52}}{2}$
$DAX = \sin^{-1}\left(\frac{\sqrt{52}}{20}\right) = 21.1°$

b i $CE = DB = \sqrt{52}$
$\sqrt{52}\cos CED = 6$
$CED = \cos^{-1}\left(\frac{6}{\sqrt{52}}\right) = 33.7°$

ii $10\cos DBA = \frac{\sqrt{52}}{2}$
$\text{Angle } DBA = \cos^{-1}\left(\frac{\sqrt{52}}{20}\right) = 68.9$

6 a i $CE^2 = 8^2 + 3^2 = 73$
$CE = \sqrt{73} = 8.5\,\text{cm}$

ii $9\sin CAX = \left(\frac{\sqrt{73}}{2}\right)$
$CAX = \sin^{-1}\left(\frac{\sqrt{73}}{18}\right) = 28.3$

b i $8\tan BDE = 3$
$BDE = \tan^{-1}\left(\frac{3}{8}\right) = 20.6$

ii $9\cos D = \frac{\sqrt{73}}{2}$
$D = \cos^{-1}\left(\frac{\sqrt{73}}{18}\right) = 61.7$

7 a $XY = 13\cos 60 = 6.5\,\text{cm}$

b $YZ = 13\sin 60 = 11.3\,\text{cm}$

c $\text{Circumference} = 2\pi \times YZ = 70.7\,\text{cm}$

8 a $XZ\cos 40 = 9$
$XZ = 9 \div \cos 40 = 11.7\,\text{cm}$

b $9\tan 40 = XY = 7.6\,\text{cm}$

9 a $PR^2 = RS^2 + SP^2 = 19.2^2 + 16^2$
$PR = \sqrt{19.2^2 + 16^2} = 25.0\,\text{cm}$

b $RV^2 = RS^2 + VS^2 = 19.2^2 + 7.2^2$
$RV = \sqrt{19.2^2 + 7.2^2} = 20.5\,\text{cm}$

c $PW^2 = PR^2 + RW^2 = 19.2^2 + 16^2 + 7.2^2$
$PW = \sqrt{19.2^2 + 16^2 + 7.2^2}$
$PW = 26.0\,\text{cm}$

d $XY^2 = XW^2 + WY^2 = \left(\frac{19.2}{2}\right)^2 + 8^2$
$XY = \sqrt{\left(\frac{19.2}{2}\right)^2 + 8^2}$
$XY = 12.5\,\text{cm}$

e $SY^2 = SW^2 + WY^2 = 20.5^2 + 8^2$
$SY = \sqrt{20.5^2 + 8^2} = 22.0\,\text{cm}$

10 a $QT^2 = 8^2 + 6^2 = 100$
$QT = 10\,\text{cm}$
$TU^2 = (10 - 4)^2 + 8^2 = 6^2 + 8^2 = 100$
$TU = 10\,\text{cm}$
$QU^2 = 6^2 + 6^2$
$QU = \sqrt{72} = 8.5\,\text{cm}$

b Using the cosine rule,
$10^2 = 10^2 + 72 - (2 \times 10 \times \sqrt{72}\cos U)$
$\cos U = \frac{72}{20\sqrt{72}} = \frac{\sqrt{72}}{20}$
$\text{Angle } U = \cos^{-1}\left(\frac{\sqrt{72}}{20}\right) = 64.9°$
As QTU is isosceles, angle $Q = 64.9°$
So angle $T = 180 - (2 \times 64.9) = 50.2°$

c Area of QTU = $\frac{1}{2} \times QT \times TU \sin T$
$= \frac{1}{2} \times 10 \times 10 \times \sin 50.2° = 38.4\,\text{cm}^2$

Exercise 5.4.2

4 a $BH^2 = 3^2 + 4^2 + 3^2 = 34$
$BH = \sqrt{34} = 5.83\,\text{cm}$

b $\sin BHF = \frac{3}{\sqrt{34}}$

Angle BHF $= \sin^{-1}\left(\frac{3}{\sqrt{34}}\right) = 31.0°$

5 a $AG^2 = 4^2 + 5^2 + 8^2 = 105$
$AG = \sqrt{105} = 10.2\,\text{cm}$

b $\sin EGA = \frac{5}{\sqrt{105}}$

Angle EGA $= \sin^{-1}\left(\frac{5}{\sqrt{105}}\right) = 29.2°$

c $\sin HAG = \frac{8}{\sqrt{105}}$

Angle HAG $= \sin^{-1}\left(\frac{8}{\sqrt{105}}\right) = 51.3°$

6 a $BD^2 = 6^2 + 3^2 = 45$
$BD = \sqrt{45} = 6.71$

b $\cos DBA = \frac{0.5 \times \sqrt{45}}{7}$

Angle DBA $= \cos^{-1}\left(\frac{\sqrt{45}}{14}\right) = 61.4°$

7 a $WY^2 = 6^2 + 5^2 = 61$
$WY = \sqrt{61} = 7.81$

b $UX^2 = 12^2 - (0.5WY)^2 = 12^2 - (0.5\sqrt{61})^2$
$UX = 11.3$

c $\tan\theta = \frac{2.5}{11.3}$

$\theta = \tan^{-1}\frac{2.5}{11.3} = 12.5°$

8 a $DB^2 = 10^2 + 10^2 = 200$
$DB = \sqrt{200} = 10\sqrt{2} = 14.1$
$HF^2 = 6^2 + 6^2 = 72$
$HF = \sqrt{72} = 6\sqrt{2} = 8.49$

c Construct a perpendicular from H to the diagonal DB.
Using **a** and **b**, distance from perpendicular along DB to D is $0.5 \times (10\sqrt{2} - 6\sqrt{2}) = 2\sqrt{2}$
So $(2\sqrt{2})^2 + h^2 = 8^2$ where h is the vertical height
$h^2 = 64 - 8 = 56$
$h = \sqrt{56} = 7.48\,\text{cm}$

d Angle DH makes with the plane ABCD
$= \sin^{-1}\left(\frac{7.48}{8}\right) = 69.2°$

9 a $AC^2 = 12^2 + 12^2 = 288$
$AC = \sqrt{288} = 12\sqrt{2} = 17.0\,\text{cm}$

b $EG^2 = 4^2 + 4^2 = 32$
$EG = \sqrt{32} = 4\sqrt{2} = 5.66$

c Construct a perpendicular from E to the diagonal AC.
Using **a** and **b**, distance from perpendicular along AC to A is $0.5 \times \left(12\sqrt{2} - 4\sqrt{2}\right) = 4\sqrt{2}$
So $(4\sqrt{2})^2 + h^2 = 9^2$, where h is the vertical height
$h^2 = 81 - 32 = 49$
$h = 7.00\,\text{cm}$

d Angle CG makes with the plane EFGH
$= \sin^{-1}\left(\frac{7}{9}\right) = 51.1°$

Exercise 5.5.1

1 a Volume $= 6 \times 2.3 \times 2 = 27.6\,\text{cm}^3$
Surface area $= (2 \times 6 \times 2.3) + (2 \times 6 \times 2) + (2 \times 2.3 \times 2)$
$= 27.6 + 24 + 9.2 = 60.8\,\text{cm}^2$

b Volume $= \pi r^2 h = 277.1\,\text{cm}^3$
Surface area $= 2\pi r(r + h)$
$= 2 \times \pi \times 3.5 \times (3.5 + 7.2)$
$= 235.3\,\text{cm}^2$

c Volume $= \frac{1}{2} \times 5 \times 2.4 \times 7 = 42\,\text{cm}^3$
Surface area
$= \left(2 \times \frac{1}{2} \times 5 \times 2.4\right) + (5 \times 7) + (2 \times 3.46 \times 7)$
$= 95.4\,\text{cm}^2$

2 a Height of cube $= 2 \times 8 = 16\,\text{cm}$

b Volume $= 16^3 = 4096\,\text{cm}^3$

c Volume of cylinder $= \pi r^2 \times 16 = 3217\,\text{cm}^3$

d % volume of cube occupied by cylinder =
$\frac{3217}{4096} \times 100\% = 78.5\%$
So % volume not occupied = 21.5%

3 a cross-sectional area $= 6 \times \frac{1}{2} \times 4 \times 3.5$
$= 42\,\text{cm}^2$

b volume $= 42 \times 20 = 840\,\text{cm}^3$

c Surface area $= (6 \times 4 \times 20) + (2 \times 42)$
$= 564\,\text{cm}^2$

4 Equating volumes: $\pi \times 2.5^2 \times 8 = 5 \times 5 \times h$
$h = \dfrac{\pi \times 2.5^2 \times 8}{5 \times 5} = 6.3\,\text{cm}$

5 Volume steel used
$= (\pi \times (0.36)^2 \times 130) - (\pi \times (0.35)^2 \times 130)$
$= 130 \times \pi \times (0.36^2 - 0.35^2)$
$= 2.90\,\text{m}^3$

6 a Surface area (large) $= 6 \times 4 \times 4$
$= 4 \times$ Surface area (small)
So Surface area (small) $= 24\,\text{cm}^2$
b In small cube surface area $= 6 \times$ area of one face $= 24\,\text{cm}^2$
So area of one face $= 4\,\text{cm}^2$
So length of each edge $= \sqrt{4} = 2\,\text{cm}$

7 a Surface area of cube = 6 faces of 6×6
$= 216\,\text{cm}^2$
b Surface area of cylinder
$= (2 \times \pi \times 2^2) + (2 \times \pi \times 2 \times h) = 216$
$h = \dfrac{216 - 8\pi}{4\pi} = 15.2\,\text{cm}$
c Difference in volume $= 6^3 - (\pi \times 2^2 \times 15.2)$
$= 25.0\,\text{cm}^3$

8 a Surface area of shorter cylinder
$= 2\pi \times 3(3 + 2) = 30\pi = 94.2\,\text{cm}^2$
b Surface area of taller cylinder
$= 2\pi \times 1(1 + h) = 30\pi$
$h = \dfrac{30\pi - 2\pi}{2\pi} = 14\,\text{cm}$
c Difference in volume
$= (\pi \times 3^2 \times 2) - (\pi \times 1^2 \times 14)$
$= 18\pi - 14\pi = 4\pi = 12.6\,\text{cm}^3$

9 Surface area
$= (2 \times 3 \times 4) + (2 \times 4 \times 2) + (2 \times 3 \times 2)$
$= 24 + 16 + 12 = 52\,\text{cm}^2$
Surface area
$= (2 \times 1 \times 4) + (2 \times 1 \times h) + (2 \times 4 \times h)$
$= 8 + 10h = 52\,\text{cm}^2$
So $10h = 44$, i.e. $h = 4.4\,\text{cm}$

Exercise 5.5.2

1 a Volume $= \frac{4}{3}\pi \times 6^3 = 905\,\text{cm}^3$
Surface area $= 4\pi \times 36 = 452\,\text{cm}^2$
b Volume $= \frac{4}{3}\pi \times 9.5^3 = 3591\,\text{cm}^3$
Surface area $= 4\pi \times 9.5^2 = 1134\,\text{cm}^2$
c Volume $= \frac{4}{3}\pi \times 8.2^3 = 2309.6\,\text{cm}^3$
Surface area $= 4\pi \times 8.2^2 = 845\,\text{cm}^2$
d Volume $= \frac{4}{3}\pi \times (0.7)^3 = 1.4\,\text{cm}^3$
Area $= 4\pi \times (0.7)^2 = 6.16\,\text{cm}^2$

2 a $\frac{4}{3}\pi r^3 = 720\,\text{cm}^3$
$r = 5.6\,\text{cm}$
b $\frac{4}{3}\pi r^3 = 0.2$
$r = 0.36 = 0.4\,\text{m}$ (1d.p.)

3 a $4\pi r^2 = 16.5$
$r = \sqrt{\dfrac{16.5}{4\pi}} = 1.15\,\text{cm}$
b $4\pi r^2 = 120\,\text{mm}^2$
$r = \sqrt{\dfrac{30}{\pi}} = 3.09\,\text{mm}$

4 Volume of sphere B $= \frac{4}{3}\pi r^3 = 2 \times$ volume of sphere A $= \frac{4}{3}\pi \times 5^3$
$r^3 = 250$
$r = \sqrt[3]{250} = 6.30\,\text{cm}$

5 Volume of outer hemisphere $= \frac{2}{3}\pi \times 5.5^3$
Volume of inner hemisphere $= \frac{2}{3}\pi \times 5^3$
So material used $= \frac{2}{3}\pi\,(5.5^3 - 5^3) = 86.7\,\text{cm}^3$

6 $\frac{2}{3}\pi(r^3 - 10^3) = \frac{4}{3}\pi \times 7^3$
$r^3 = 2 \times 7^3 + 1000$
$r = 11.9\,\text{cm}$

7 a Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 10^3 = 4190\,\text{cm}^3$
b Volume $= (2 \times 10)^3 = 8000\,\text{cm}^3$
c % occupied by ball $= \frac{4190}{8000} \times 100 = 52.4\%$
So % not occupied by ball = 47.6%

8 Volume of original ball $= \frac{4}{3}\pi \times 20^3$
Now $\frac{4}{3}\pi \times 20^3 = 8 \times \frac{4}{3}\pi r^3$ where r is the radius of the smaller ball
So $r^3 = \dfrac{20^3}{2^3}$, giving $r = 10\,\text{cm}$

9 Volume of ball A $= \frac{7}{15} \times 600 = \frac{4}{3}\pi r_A^3$
So $r_A = 4.1\,\text{cm}$
Volume of ball B $= \frac{5}{15} \times 600 = \frac{4}{3}\pi r_B^3$
So $r_B = 3.6\,\text{cm}$
Volume of ball C $= \frac{3}{15} \times 600 = \frac{4}{3}\pi r_C^3$
So $r_C = 3.1\,\text{cm}$

10 Volume (cylinder) $= \pi r^2 \times 2r = 2\pi r^3$
Volume (sphere) $= \frac{4}{3}\pi r^3$
Volume (cylinder) : Volume (sphere)
$= 2 : \frac{4}{3}$
$= 6 : 4$
$= 3 : 2$

11 Surface area (A) $= 4\pi \times 8^2$
Surface area (B) $= 4\pi \times 16^2$
Surface area (A) : Surface area (B) $= 8^2 : 16^2$,
i.e., $8^2 : (2 \times 8)^2 = 1 : 4$

12 **a** Surface area (hemisphere) $= \frac{1}{2} \times 4\pi \times 5^2$
$= 157\,\text{cm}^2$
b Length of cylinder $= 20 - 5 = 15\,\text{cm}$
c $157 + (\pi \times 5^2) + (2\pi \times 5 \times 15)$
$= 157 + 25\pi + 150\pi$
$= 707\,\text{cm}^2$

13 **a** Surface area (sphere) $= 4\pi \times 8^2 = 256\pi$
$= 804\,\text{cm}^2$
b Surface area (cylinder) = surface area (sphere)
$(2 \times \pi r^2) + (2\pi r \times 16) = 256\pi$
$r^2 + 16r - 128 = 0$
Using the quadratic formula
$$r = \frac{-16 \pm \sqrt{16^2 + 512}}{2}$$
and taking the positive root $r = 5.9\,\text{cm}$

Exercise 5.5.3

1 **a** Volume $= \frac{1}{3} \times \text{base} \times h = \frac{1}{3} \times 5 \times 4 \times 6$
$= 40\,\text{cm}^3$
b Volume $= \frac{1}{3} \times 50 \times 8 = 133\,\text{cm}^3$
c Area of base $= \frac{1}{2} \times 6 \times \sqrt{10^2 - 6^2} = 24$
Volume $= \frac{1}{3} \times 24 \times 8 = 64\,\text{cm}^3$
d Volume $= \frac{1}{3} \times 6 \times 7 \times 5 = 70\,\text{cm}^3$

2 Diagonal$^2 = 5^2 + 8^2$
Length of diagonal $= \sqrt{25 + 64} = \sqrt{89}$
$$12^2 = h^2 + \left(\frac{\sqrt{89}}{2}\right)^2$$
$h^2 = 144 - \frac{89}{4}$
$h = 11.03$
Volume $= \frac{1}{3} \times 8 \times 5 \times h = 147.1\,\text{cm}^3$
Surface area $= (8 \times 5) + (2 \times 0.5 \times 5 \times h_1) + (2 \times 0.5 \times 8 \times h_2)$
$$h_1 = \sqrt{12^2 - \left(\frac{5}{2}\right)^2}$$
$h_1 = 11.74$
$h_2 = \sqrt{12^2 - 4^2}$
$h_2 = 11.31$
Surface area = 189.2

3 Volume of two pyramids $= \frac{2}{3} \times 4 \times 4 \times \text{height}$
$$h^2 = 5^2 - \left(\frac{\sqrt{32}}{2}\right)^2 = 25 - 8$$
$h = \sqrt{17}$
Volume of two pyramids $= \frac{2}{3} \times 4 \times 4 \times \sqrt{17}$
$V = 44.0\,\text{cm}^3$
Surface area of one face $= \frac{1}{2} \times 4 \times \sqrt{21}$
Total surface area $= 16\sqrt{21} = 73.3\,\text{cm}^2$

4 **a** As edge length of small pyramid is $\frac{1}{4}$ original, vertical height of small pyramid will be in the same proportion to that of the original.
So height of small pyramid $= \frac{1}{4}$ height of original. Let x be the height of the original:
$6 + \frac{1}{4}x = x$
$6 = \frac{3}{4}x$
$x = 8\,\text{cm}$
b Volume $= \frac{1}{3} \times 12 \times 12 \times 8 = 384\,\text{cm}^3$
c Volume (small) $= \frac{1}{3} \times 3 \times 3 \times 2 = 6\,\text{cm}^3$
Volume (truncated) $= 384 - 6 = 378\,\text{cm}^3$

5 Surface area $= (18 \times 18) + (9 \times 9) +$ (4 × surface area of each side of trapezium)
Height of trapezium $= \sqrt{14^2 - 4.5^2}$
Area of trapezium $= \frac{18 + 9}{2} \times \sqrt{14^2 - 4.5^2} = 178.97$
Surface area $= (18 \times 18) + (9 \times 9) + (4 \times 178.97)$
$= 324 + 81 + 715.9$
$= 1120.9\,\text{cm}^2$

6 $168 = \frac{1}{3} \times 9 \times 8 \times h$
$$h = \frac{168 \times 3}{9 \times 8} = 7\,\text{cm}$$

7 $14 = \frac{1}{3} \times \frac{3x}{2} \times 7$
$x = 4\,\text{cm}$

8 **a** Height of original $= \frac{8}{5} \times 6 = 9.6$
Height of truncated piece $= 9.6 - 6 = 3.6\,\text{cm}$

b Volume $= \frac{1}{3} \times$ base area $\times 6$

base area $= \frac{1}{2} \times 5 \times 5 \sin 60°$

$= \frac{1}{2} \times 5 \times 5 \times \frac{\sqrt{3}}{2}$

Volume $= \frac{1}{3} \times \frac{1}{2} \times 5 \times 5 \times \frac{\sqrt{3}}{2} \times 6$

$= \frac{25\sqrt{3}}{2} = 21.7\,\text{cm}^3$

c Volume original $= \frac{1}{3} \times \frac{1}{2} \times 8 \times 8 \times \frac{\sqrt{3}}{2} \times 9.6$
$= 88.7\,\text{cm}^3$

9 Surface area of one face
$= \frac{1}{2} \times 2 \times 2 \sin 60° = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$
Total surface area $= 4\sqrt{3} = 6.93\,\text{cm}^2$

10 a Area of one face in
A $= \frac{1}{2} \times 20 \times 20 \times \sin 60° = 200 \times \frac{\sqrt{3}}{2}$

Surface area of tetrahedron $= 4 \times 200 \times \frac{\sqrt{3}}{2}$
$= 693\,\text{cm}^2$

b Surface area of A = surface area of B
$693 = 12^2 + 4TF$ where TF is the area of one of the triangular faces on the square-based pyramid $TF = 137.25\,\text{cm}^2$

c $137.25 = \frac{1}{2} \times 12 \times \sqrt{x^2 - 6^2}$

$\sqrt{x^2 - 6^2} = \frac{137.25}{6}$

$x = 23.6$ cm

Exercise 5.5.4

1 a Length of arc $= \frac{45}{360} \times 2\pi \times 8 = 6.3\,\text{cm}$

Area of sector $= \frac{45}{360} \times \pi \times 8^2 = 25.1\,\text{cm}^2$

b Length of arc $= \frac{8}{360} \times 2\pi \times 15 = 2.1\,\text{cm}$

Area of sector $= \frac{8}{360} \times \pi \times 15^2 = 15.7\,\text{cm}^2$

c Length of arc $= \frac{110}{360} \times 2\pi \times 6 = 11.5\,\text{cm}$

Area of sector $= \frac{110}{360} \times \pi \times 6^2 = 34.6\,\text{cm}^2$

d Length of arc $= \frac{45}{360} \times 2\pi \times 8 = 6.3\,\text{cm}$

Area of sector $= \frac{270}{360} \times \pi \times 5^2 = 58.9\,\text{cm}^2$

2 a Length of arc $= \frac{24}{360} \times 2\pi \times 10 = 4.19\,\text{cm}$

b Total surface area $= (4.19 \times 3) +$
$(2 \times 10 \times 3) + (2 \times \frac{24}{360} \times \pi \times 10^2$
$= 114\,\text{cm}^2$

c Volume of slice $= \frac{24}{360} \times \pi \times 10^2 \times 3 = 62.8\,\text{cm}^3$

3 a Volume $= \frac{1}{3} \times \pi \times 3^2 \times 6 = 56.5\,\text{cm}^3$

b Volume $= \frac{1}{3} \times \pi \times 6^2 \times 7 = 264\,\text{cm}^3$

c Volume $= \frac{1}{3} \times \pi \times (0.8)^2 \times 2 = 1.34\,\text{cm}^3$

d Volume $= \frac{1}{3} \times \pi \times 6^2 \times 4.4 = 166\,\text{cm}^3$

4 $600 = \frac{1}{3} \times \pi \times r^2 \times 12$

$r^2 = \frac{600}{4\pi}$

$r = \sqrt{\frac{150}{\pi}} = 6.91\,\text{cm}$

5 a Base radius $= \frac{100}{2\pi} = 15.9\,\text{cm}$

b $18^2 = 15.92 + h^2$ $h = \sqrt{18^2 - 15.9^2} = 8.41\,\text{cm}$

c Volume $= \frac{1}{3}\pi \times \left(\frac{100}{2\pi}\right)^2 \times 8.41 = 2230\,\text{cm}^3$

6 a Vertical height $= \sqrt{16^2 - 6^2} = 14.83$
Volume $= \frac{1}{3} \times \pi \times 6^2 \times 14.83 = 559.2\,\text{cm}^3$
Surface area $= (\pi + 6^2) + (\pi \times 6 \times 16)$
$= 414.7\,\text{cm}^2$

b $h = \sqrt{20^2 - 15^2} = \sqrt{175}$
Volume $= \frac{1}{3} \times \pi \times 15^2 \times \sqrt{175} = 3117.0\,\text{cm}^3$
Surface area $= (\pi \times 15^2) + (\pi \times 15 \times 20$
$= 525\pi = 1649.3$

7 Height of cone A
$= \sqrt{15^2 - 5^2} = \sqrt{200} = 10\sqrt{2}$ cm
Base circumference of cone B is 60 cm.
So radius of base $= \frac{60}{2\pi} = \frac{30}{\pi}$
Volume of cone A = volume of cone B
$\frac{1}{3} \times \pi \times 5^2 \times 10\sqrt{2} = \frac{1}{3} \times \pi \times \left(\frac{30}{\pi}\right)^2 \times h$

$h = \frac{250\pi^2\sqrt{2}}{30^2} = 3.88\,\text{cm}$

8 a Base circumference $= \frac{210}{360} \times 2\pi \times 9 = 33.0\,\text{cm}$

b Base radius = circumference $\div 2\pi$
$= \frac{210}{360} \times 9 = \frac{21}{4} = 5.2\,\text{cm}$

c Vertical height = $\sqrt{9^2 - 5.25^2} = 7.31$

d Volume of cone = $\frac{1}{3} \times \pi \times 5.25^2 \times 7.31$
$= 211\,\text{cm}^3$

e Curved surface area = $\pi \times 5.25 \times 9 = 148\,\text{cm}^2$

9 Surface area = $(\pi \times 8 \times 15) + (\pi \times 8 \times 30)$
$= 360\pi = 1131\,\text{cm}^2$

10 a Total surface area of first cone
$= (\pi \times 5^2) + (\pi \times 5 \times 15) = 100\pi = 314\,\text{cm}^4$

b Total surface area of second cone =
$= (\pi \times 8^2) + (\pi \times 8 \times x) = 100\pi$
$64\pi + 8\pi x = 100\pi$
$8x = 36$
$x = 4.5$

11 a Volume of cuboid = $12 \times 12 \times 16 = 2304\,\text{cm}^3$

b Volume of cone = $\frac{1}{3} \times \pi \times 6^2 \times 16$
$= 603.2\,\text{cm}^3$

c Volume not occupied = $2304 - 603.2$
$= 1700.8\,\text{cm}^3$

12 a Radius of smaller cone = $\frac{260}{360} \times 6 = \frac{13}{3}$

Height of smaller cone
$= \sqrt{6^2 - \left(\frac{13}{3}\right)^2} = \frac{\sqrt{155}}{3}$

Volume of smaller cone
$= \frac{1}{3} \times \pi \times \left(\frac{13}{3}\right)^2 \times \frac{\sqrt{155}}{3} = 81.6$

b Radius of larger cone = $\frac{260}{360} \times 9 = \frac{13}{2}$

Height of larger cone = $\sqrt{9^2 - \left(\frac{13}{2}\right)^2} = \frac{\sqrt{155}}{2}$

Volume of larger cone
$= \frac{1}{3} \times \pi \times \left(\frac{13}{2}\right)^2 \times \frac{\sqrt{155}}{2} = 275.4$

c Ratio of volumes is
$\left(\frac{13}{3}\right)^2 \times \frac{\sqrt{155}}{3} : \left(\frac{13}{2}\right)^2 \times \frac{\sqrt{155}}{2}$
$\frac{1}{3^3} : \frac{1}{2^3} = 8:27$

13 a Height of cone = $10 - 2.5 = 7.5\,\text{cm}$
Total volume
= volume of cone + volume of hemisphere
$= \left(\frac{1}{3} \times \pi \times 2.5^2 \times 7.5\right) + \left(\frac{2}{3}\pi \times 2.5^2\right) = 81.8\,\text{cm}^3$

b Slant height of cone = $\sqrt{7.5^2 + 2.5^2}$
Total surface area = surface area of cone + surface area of hemisphere
$= (\pi \times 2.5 \times \sqrt{7.5^2 + 2.5^2} +$
$(2\pi \times 2.5^2) = 101\,\text{cm}^2$

14 a Total volume = volume of cone + volume of cylinder
$= \left(\frac{1}{3} \times \pi \times 4^2 \times 10\right) + (\pi \times 4^2 \times 12) = 771\,\text{cm}^3$

b Total surface area = bottom + cylinder + cone
$= (\pi \times 4^2) + (2\pi \times 4 \times 12) +$
$(\pi \times 4 \times \sqrt{10^2 + 4^2})$
$= 112\pi + 4\pi \times \sqrt{116} = 487\,\text{m}^2$

15 Consider a full cone with base 16 cm.
Height of full cone = $18 + x$
Using similar triangles $\frac{18 + x}{8} = \frac{x}{2}$
i.e. $18 + x = 4x$
$3x = 18$
$x = 6$
Volume of truncated cone = volume of full cone – volume of small cone (height 6)
$= \left(\frac{1}{3} \times \pi \times 8^2 \times 24\right) - \frac{1}{3} \times (\pi \times 2^2 \times 6) =$
$1583.36\,\text{cm}^3$

Volume of shape = $2 \times 1583.35 = 3166.7\,\text{cm}^3$

16 a Volume A = $\frac{1}{3} \times \pi \times 5^2 \times 25 = 654.5\,\text{cm}^3$

b Volume A : Volume B = 2 : 1
So volume B = $327.25 = \frac{1}{3} \times \pi \times 5^2 \times h$
Giving $h = 12.5\,\text{cm}$

c Volume of cylinder = $\pi \times 5^2 \times (25 + 12.5)$
$= 2945.2\,\text{cm}^3$

Student assessment 1 (Topic 5)

1 a $(-6, -1)$ $(6, 4)$

i Distance between points
$= \sqrt{12^2 + 5^2} = 13$ units

ii Coordinate of midpoint is
$\left(\frac{-6+6}{2}, \frac{-1+4}{2}\right) = (0, \frac{3}{2})$

b $(1, 2)$ $(7, 10)$

i Distance between points
$= \sqrt{6^2 + 8^2} = 10$ units

ii Coordinate of midpoint is
$\left(\frac{1+7}{2}, \frac{2+10}{2}\right) = (4, 6)$

c (2, 6) (–2, 3)

i Distance between points

$= \sqrt{4^2 + 3^2} = 5$ units

ii Coordinate of midpoint is

$\left(\frac{2-2}{2}, \frac{6+3}{2}\right) = (0, 4.5)$

d (–10, –10) (0, 14)

i Distance between points

$= \sqrt{10^2 + 24^2} = 26$ units

ii Coordinate of midpoint is

$\left(\frac{-10+0}{2}, \frac{-10+14}{2}\right) = (-5, 2)$

3 a i $y = x + 1$

$m = 1, c = 1$

b i $y = 3 - 3x$

$m = -3, c = 3$

c i $2x - y = -4$, i.e. $y = 2x + 4$

$m = 2, c = 4$

d i $2y - 5x = 8$

$m = \frac{5}{2}, c = 4$

4 a (1, –1) (4, 8)

Gradient $= \frac{8 - -1}{4 - 1} = \frac{9}{3} = 3$

Line passes through (1, –1)

$-1 = (3 \times 1) + c$, giving $c = -4$

Equation is $y = 3x - 4$

b (0, 7) (3, 1)

Gradient $= \frac{7 - 1}{0 - 3} = \frac{6}{-3} = -2$

Line passes through (0, 7)

$7 = (0 \times 2) + c$, giving $c = 7$

Equation is $y = -2x + 7$

c (–2, –9) (5, 5)

Gradient $= \frac{5 - -9}{5 - -2} = \frac{14}{7} = 2$

Line passes through (5, 5)

$5 = (5 \times 2) + c$, giving $c = -5$

Equation is $y = 2x - 5$

d (1, –1) (–1, 7)

Gradient $= \frac{7 - -1}{-1 - 1} = \frac{8}{-2} = -4$

Line passes through (1, –1)

$-1 = (-4 \times 1) + c$, giving $c = 3$

Equation is $y = -4x + 3$

5 a $x + y = 4$ (1)

$x - y = 0$ (2)

(1) + (2): $2x = 4$, i.e. $x = 2$

In (1) $2 + y = 4$, i.e. $y = 2$

b $3x + y = 2$ (1)

$x - y = 2$ (2)

(1) + (2): $4x = 4$, i.e. $x = 1$

In (2) $1 - y = 2$, i.e. $y = -1$

c $y + 4x + 4 = 0$ (1)

$x + y = 2$ (2)

(1) – (2): $3x + 4 = -2$, i.e. $x = -2$

In (2) $-2 + y = 2$, i.e. $y = 4$

d $x - y = -2$ (1)

$3x + 2y + 6 = 0$ (2)

(1) × 2: $2x - 2y = -4$ (3)

(2) + (3): $5x + 6 = -4$, i.e. $x = -2$

In (1) $-2 - y = -2$, i.e. $y = 0$

Student assessment 2 (Topic 5)

1 a $\sin 30° = \frac{x}{8}$, i.e. $x = 8 \sin 30 = 4.0$ cm

b $\sin 20° = \frac{15}{x}$, i.e. $x = \frac{15}{\sin 20} = 43.9$ cm

c $\cos 60° = \frac{10.4}{x}$, i.e. $x = \frac{10.4}{\cos 60°} = 20.8$ cm

d $\sin 50° = \frac{3}{x}$, i.e. $x = \frac{3}{\sin 50°} = 3.9$ cm

2 a $\theta = \sin^{-1}\left(\frac{9}{15}\right) = 37°$

b $\theta = \tan^{-1}\left(\frac{6.3}{4.2}\right) = 56°$

c $\theta = \tan^{-1}\left(\frac{3}{5}\right) = 31°$

d $\theta = \cos^{-1}\frac{12.3}{14.8} = 34°$

3 a $q = \sqrt{3^2 + 4^2} = 5.0$

b $q = \sqrt{12^2 - 10^2} = \sqrt{44} = 6.6$

c Length of common line is $\frac{3}{\cos 65°}$

$q = \sqrt{\left(\frac{3}{\cos 65°}\right)^2 + 6^2} = 9.3$ cm

Length of base is $\frac{18}{\tan 25°}$

$q = \sqrt{48^2 - \left(\frac{18}{\tan 25°}\right)^2} = 28.5$ cm

Student assessment 3 (Topic 5)

1 a $\tan 25° = \dfrac{75}{AB}$

$AB = \dfrac{75}{\tan 25°} = 160.8\,\text{km}$

b $BC^2 = \sqrt{AB^2 + 75^2} = 177.5\,\text{km}$

2 a $\tan\theta° = \dfrac{5}{x}$

b $\tan\theta° = \dfrac{7.5}{(x+16)}$

c $\dfrac{5}{x} = \dfrac{7.5}{(x+16)}$

d $5(x + 16) = 7.5x$

$2.5x = 80$

$x = 32$

e $\theta = \tan^{-1}\frac{5}{32} = 8.88°$

3 a $BY = \sqrt{320^2 - 145^2} = 285\,\text{m}$

b $\sin BYX = \frac{145}{320}$

$BYX = \sin^{-1}\frac{145}{320} = 27°$

So bearing of Y from X = 90 + 27 = 117°

c Bearing of X from Y = 270 + 27 = 297°

4 a Height of P above ground $= 2.8\tan 35°$
$= 1.96\,\text{km}$

b $PR = \dfrac{2.8}{\cos 35°} = 3.42\,\text{km}$

c $QR^2 = 2.8^2 + (1.96 + 0.25)^2$

$QR = \sqrt{2.8^2 + 2.21^2} = 3.57\,\text{km}$

5 a $AB^2 = 4000^2 + 164^2$

$AB = \sqrt{4000^2 + 164^2} = 4003\,\text{km}$

b $\text{Angle} = \tan^{-1}\frac{164}{4000} = 2.35°$

7 a $\sin 50° = \sin(180 - 50) = \sin 130°$

b $\sin 150° = \sin(180 - 150) = \sin 30°$

c $\cos 45° = -\cos(180 - 45) = -\cos 135°$

d $\cos 120° = -\cos(180 - 120) = -\cos 60°$

8 Using the sine rule $\dfrac{\sin\theta}{26} = \dfrac{\sin 30°}{18}$

$\theta = \sin^{-1}\left(\dfrac{26\sin 30°}{18}\right) = 46°$ or $(180 - 46)°$

As angle is obtuse $\theta = 134°$

Student assessment 4 (Topic 5)

1 a $EG^2 = 10^2 + 6^2$

$EG = \sqrt{10^2 + 6^2} = 11.7\,\text{cm}$

b $EC^2 = 10^2 + 6^2 + 4^2$

$EC = \sqrt{10^2 + 6^2 + 4^2} = 12.3\,\text{cm}$

c $BE = \sqrt{10^2 + 4^2}$

$\text{Angle BEC} = \cos^{-1}\dfrac{BE}{EC} = 29°$

2 a $AD = \sqrt{6^2 + 9^2} = 10.8\,\text{cm}$

b $AC = \sqrt{6^2 + 9^2 + 5^2} = 11.9$

c Let θ be the angle that AC makes with the plane CDEF

$\sin\theta = \dfrac{6}{AC}$

$\theta = \sin^{-1}\dfrac{6}{11.9} = 30°$

d Let ϕ be the angle AC makes with the plane ABFE

$\sin\phi = \dfrac{9}{AC}$

$\phi = \sin^{-1}\dfrac{9}{11.9} = 49°$

4 a $\cos 128° = -\cos(180 - 128) = -\cos 52°$

b $-\cos 80° = \cos(180 - 80) = \cos 100°$

5 a In the isosceles triangle acute angles are $\frac{1}{2}(180 - 120) = 30°$

Using the sine rule $\dfrac{PS}{\sin 30°} = \dfrac{17}{\sin 120°}$

b Angle QRS = 180 – 90 – 60 = 30°

$\cos 60° = \dfrac{9.8}{SR}$

$SR = \dfrac{9.8}{\cos 60°} = 19.6$

6 a Let b be the distance from B to the centre of the base of the pyramid.

$\tan 12° = \dfrac{146 - 1.8}{b}$

$b = \dfrac{144.2}{\tan 12°} = 678.4\,\text{m}$

b A is 678.4 + 25 = 703.4 m from the centre of the base of the pyramid.

$\tan\theta = \dfrac{144.2}{703.4}$

$\theta = \tan^{-1}\dfrac{144.2}{703.4} = 11.6°$

c Let a be the distance between A and the top of the pyramid

$\sin 11.585° = \frac{144.2}{a}$

$a = \frac{144.2}{\sin 11.585°} = 718.1$ m

Student assessment 5 (Topic 5)

1 a Using the cosine rule,
$JL^2 = 25^2 + 12^2 - (2 \times 25 \times 12 \cos 42°)$
JL = 18.0 m (to 1 d.p.)

b Using the sine rule,
$\frac{\sin KJL}{12} = \frac{\sin 42°}{18.0}$
Angle KJL $= \sin^{-1}\left(\frac{12\sin 42°}{18.0}\right) = 27°$

c Angle MJL = 70 – 27 = 43°
Let JM = x and using the cosine rule,
$20^2 = x^2 + 17.98^2 -$
$(2 \times 17.98 \times x \times \cos 43.48°)$
$x^2 - 26.09x - 76.72 = 0$
So $x = \frac{26.09 \pm \sqrt{26.09^2 + 4 \times 76.72}}{2}$
$x = 26.09 \pm 31.43$
$x = \frac{26.09 \pm 31.43}{2} = 28.8$
(taking the positive root)

d Area of JKLM = area JKL + area JLM
$= (0.5 \times 25 \times 12 \sin 42°) +$
$(0.5 \times 28.8 \times 18 \sin 43°) = 277.1\text{m}^2$

2 a $BD = \sqrt{9^2 + 9^2} = \sqrt{162} = 12.7$ cm

b $\cos ABD = \frac{0.5 \times 12.7}{16}$
Angle ABD $= \cos^{-1}\left(\frac{0.5 \times 12.7}{16}\right) = 66.6 = 67°$
(to nearest degree)

c Area ABD $= 0.5 \times 16 \times \sqrt{162} \times \sin 66.6°$
$= 93.4\text{cm}^2$

d Area ABD = 0.5 × base × height
$93.4 = 0.5 \times \sqrt{162} \times$ height
Height = 14.7 cm

3 a $\cos^{-1} 0.79 = 37.8 = 38°$ and
$(360 - 38) = 322°$

b $\cos^{-1} -0.28 = (180 - 74) = 106°$ and
$(180 + 74) = 254°$

5 a $DC = \sqrt{5^2 + 3^2} = \sqrt{34} = 5.8$ cm

b $BC = \sqrt{6^2 + 3^2} = \sqrt{45} = 6.7$ cm

c $DB = \sqrt{6^2 + 5^2} = \sqrt{61} = 7.8$ cm

d Using the cosine rule,
$5.8^2 = 6.7^2 + 7.8^2 - (2 \times 6.7 \times 7.8 \cos CBD)$
Angle CBD $= \cos^{-1}\left(\frac{6.7^2 + 7.8^2 - 5.8^2}{2 \times 6.7 \times 7.8}\right) = 47°$

e Length AC $= \sqrt{5^2 + 6^2 + 3^2} = \sqrt{70}$
Angle AC makes with plane AEHD
$= \sin^{-1}\left(\frac{5}{\sqrt{70}}\right) = 37°$

6 a $PR = \sqrt{24^2 + 20^2} = 31.2$ cm

b $RV = \sqrt{24^2 + 9^2} = 25.6$ cm

c $WP = \sqrt{24^2 + 20^2 + 9^2} = 32.5$ cm

d $XY = \sqrt{12^2 + 10^2} = 15.6$ cm

e $SY = \sqrt{24^2 + 9^2 + 10^2} = 27.5$ cm

Student assessment 6 (Topic 5)

1 a Total surface area $= 4 \times \pi \times 6.5^2 = 530.9\text{cm}^2$

b Volume $= \frac{4}{3} \times \pi \times 6.5^3 = 1150.3\text{cm}^3$

2 a The regular hexagon consists of 6 equilateral triangles with side length of 12.
Area of hexagon $= 6 \times 0.5 \times 12 \times 12 \times \sin 60°$
$= 374.12\text{cm}^2$
Height of side triangle $\sqrt{24^2 - 6^2} = \sqrt{540}$
Area of 6 side triangles
$= 6 \times 0.5 \times 12 \times \sqrt{540} = 836.56\text{cm}^2$
Total surface area of pyramid
= 374.12 + 836.56 = 1210.7 (to 1 d.p.)

b Volume $= \frac{1}{3} \times$ base area × perpendicular height
Height $= \sqrt{24^2 - 12^2} = 12\sqrt{3}$
Volume $= \frac{1}{3} \times 374.12 \times 12\sqrt{3} = 2592.0\text{cm}^3$
(to 1 d.p.)

3 a $\frac{50}{360} = \frac{20}{2\pi r}$
So $r = \frac{72}{\pi} = 22.9$ cm

b Cross-sectional area (CSA) $= \frac{50}{360} \times \pi \times 22.92^2$
$= 229.2\text{cm}^2$

c Total surface area
$= (2 \times 229.22) + (2 \times 22.92 \times 8) + (20 \times 8)$
$= 985.1$

d Volume = CSA × 8 = 229.2 × 8 = 1833.6 cm^3

4 **a** Volume of sphere $= \frac{4}{3} \times \pi \times 6^3 = 904.8\,\text{cm}^3$

b Volume of cone $= \frac{1}{3} \times$ base area × height

$904.8 = \frac{1}{3} \times \pi \times r^2 \times 6$

$r^2 = \dfrac{904.8}{2\pi}$

$r = \sqrt{\dfrac{904.8}{2\pi}} = 12.0$

c $x = \sqrt{12.0^2 + 6^2} = \sqrt{180} = 13.4\,\text{cm}$

d Surface area of cone
$= (\pi \times 12^2) + (\pi \times 12 \times \sqrt{180}) = 958.2\,\text{cm}^2$

5 **a** Ratio of diameter of original cone to diameter of removed cone = 56 : 28 = 2 : 1
So ratio of height of original cone : height of removed cone = 2 : 1
If height of remaining truncated cone = 50 mm, height of original cone = 100 mm = 10 cm

b Volume of original cone
$= \frac{1}{3} \times \pi \times 2.8^2 \times 10 = 82.1\,\text{cm}^3$

c Volume of truncated cone = volume original – volume of removed cone
$= 82.1 - \frac{1}{3} \times \pi \times 1.4^2 \times 5 = 71.8\,\text{cm}^3$

d Volume of cylindrical hole $= \pi \times 1.4^2 \times 5$
$= 30.8\,\text{cm}^3$

e Volume of remaining truncated cone
= 71.8 – 30.8 = 41.1 (using unrounded values)

Exercise 6.3.5

1 **a** $x^2 + 7x + 12 = 0$
$(x + 4)(x + 3) = 0$
$x = -4$ and $x = -3$

b $x^2 + 8x + 12 = 0$
$(x + 6)(x + 2) = 0$
$x = -6$ and $x = -2$

c $x^2 + 3x - 10 = 0$
$(x + 5)(x - 2) = 0$
$x = -5$ and $x = 2$

d $x^2 - 3x - 10 = 0$
$(x - 5)(x + 2) = 0$
$x = 5$ and $x = -2$

e $x^2 + 5x = -6$
$x^2 + 5x + 6 = 0$
$(x + 3)(x + 2) = 0$
$x = -3$ and $x = -2$

f $x^2 + 6x = -9$
$x^2 + 6x + 9 = 0$
$(x + 3)(x + 3) = 0$
$x = -3$

g $x^2 - 2x = 8$
$x^2 - 2x - 8 = 0$
$(x - 4)(x + 2) = 0$
$x = 4$ and $x = -2$

h $x^2 - x = 20$
$x^2 - x - 20 = 0$
$(x - 5)(x + 4) = 0$
$x = 5$ and $x = -4$

i $x^2 + x = 30$
$x^2 + x - 30 = 0$
$(x + 6)(x - 5) = 0$
$x = -6$ and $x = 5$

j $x^2 - x = 42$
$x^2 - x - 42 = 0$
$(x - 7)(x + 6) = 0$
$x = 7$ and $x = -6$

2 **a** $x^2 - 9 = 0$
$(x + 3)(x - 3) = 0$
$x = -3$ and $x = 3$

b $x^2 = 25$
$x^2 - 25 = 0$
$(x + 5)(x - 5) = 0$
$x = -5$ and $x = 5$

c $x^2 - 144 = 0$
$(x + 12)(x - 12) = 0$
$x = -12$ and $x = 12$

d $4x^2 - 25 = 0$
$(2x + 5)(2x - 5) = 0$
$x = -2.5$ and $x = 2.5$

e $9x^2 - 36 = 0$
$9(x^2 - 4) = 0$
$9(x + 2)(x - 2) = 0$
$x = -2$ and $x = 2$

f $x^2 - \frac{1}{9} = 0$
$\left(x + \frac{1}{3}\right)\left(x - \frac{1}{3}\right) = 0$
$x = -\frac{1}{3}$ and $x = \frac{1}{3}$

g $x^2 + 6x + 8 = 0$
$(x + 4)(x + 2) = 0$
$x = -4$ and $x = -2$

h $x^2 - 6x + 8 = 0$
$(x - 4)(x - 2) = 0$
$x = 4$ and $x = 2$

i $x^2 - 2x - 24 = 0$
$(x - 6)(x + 4) = 0$
$x = 6$ and $x = -4$

j $x^2 - 2x - 48 = 0$
$(x - 8)(x + 6) = 0$
$x = 8$ and $x = -6$

3 a $x^2 + 5x = 36$
$x^2 + 5x - 36 = 0$
$(x + 9)(x - 4) = 0$
$x = -9$ and $x = 4$

b $x^2 + 2x = -1$
$x^2 + 2x + 1 = 0$
$(x + 1)(x + 1) = 0$
$x = -1$

c $x^2 - 8x = 0$
$x(x - 8) = 0$
$x = 0$ and $x = 8$

d $x^2 - 7x = 0$
$x(x - 7) = 0$
$x = 0$ and $x = 7$

e $2x^2 + 5x + 3 = 0$
$(2x + 3)(x + 1) = 0$
$x = -1.5$ and $x = -1$

f $2x^2 - 3x - 5 = 0$
$(2x - 5)(x + 1) = 0$
$x = 2.5$ and $x = -1$

g $x^2 + 12x = 0$
$x(x + 12) = 0$
$x = 0$ and $x = -12$

h $x^2 + 12x + 27 = 0$
$(x + 9)(x + 3) = 0$
$x = -9$ and $x = -3$

i $2x^2 = 72$
$x^2 - 36 = 0$
$(x + 6)(x - 6) = 0$
$x = -6$ and $x = 6$

j $3x^2 - 12 = 288$
$3x^2 - 300 = 0$
$x^2 - 100 = 0$
$(x + 10)(x - 10) = 0$
$x = -10$ and $x = 10$

4 $x^2 + x = 12$
$x^2 + x - 12 = 0$
$(x + 4)(x - 3) = 0$
$x = -4$ and $x = 3$

5 Area $= x(x + 3) = 10$
$x^2 + 3x - 10 = 0$
$(x + 5)(x - 2) = 0$
Taking the positive value $x = 2$

6 $x(x + 9) = 52$
$x^2 + 9x - 52 = 0$
$(x + 13)(x - 4) = 0$
Taking the positive value $x = 4$

7 $\frac{1}{2} \times 2x \times (x - 3) = 18$
$x^2 - 3x - 18 = 0$
$(x - 6)(x + 3) = 0$
Taking the positive value $x = 6$
So height = 3 cm and base length = 12 cm

8 $\frac{1}{2} \times (x - 8) \times 2x = 20$
$x^2 - 8x - 20 = 0$
$(x - 10)(x + 2) = 0$
Taking the positive value $x = 10$
So height = 20 cm and base length = 2 cm

9 $\frac{1}{2} x \times (x - 1) = 15$
$x^2 - x - 30 = 0$
$(x - 6)(x + 5) = 0$
Taking the positive value $x = 6$
So base = 6 cm and height = 5 cm

10 $(7 + x)(x + 2) - x^2 = 50$
$x^2 + 9x + 14 - x^2 = 50$
$9x = 36$
$x = 4$
Garden is 11 m × 6 m

Exercise 6.3.6

1 a $x^2 - x - 13 = 0$
$x = \frac{1 \pm \sqrt{1 + 4 \times 1 \times 13}}{2} = \frac{1 \pm \sqrt{53}}{2} = 4.14$
and -3.14

b $x^2 + 4x - 11 = 0$
$x = \frac{-4 \pm \sqrt{16 + 4 \times 11}}{2} = \frac{-4 \pm \sqrt{60}}{2} = 1.87$
and -5.87

c $x^2 + 5x - 7 = 0$
$x = \frac{-5 \pm \sqrt{25 + 28}}{2} = \frac{-5 \pm \sqrt{53}}{2} = -6.14$
and 1.14

d $x^2 + 6x + 11 = 0$

$x = \dfrac{-6 \pm \sqrt{36 - 44}}{2}$ No solution

e $x^2 + 5x - 13 = 0$

$x = \dfrac{-5 \pm \sqrt{25 + 52}}{2} = -6.89$ and 1.89

f $x^2 - 9x + 19 = 0$

$x = \dfrac{9 \pm \sqrt{81 - 76}}{2} = \dfrac{9 \pm \sqrt{5}}{2} = 5.62$ and 3.38

2 a $x^2 + 7x + 9 = 0$

$x = \dfrac{-7 \pm \sqrt{49 - 36}}{2} = \dfrac{-7 \pm \sqrt{13}}{2} = -5.3$ and -1.7

b $x^2 - 35 = 0$

$x = \dfrac{0 \pm \sqrt{4 \times 35}}{2} = \pm\sqrt{35} = -5.92$ and 5.92

c $4x^2 - 20x + 25 = 0$

$x = \dfrac{20 \pm \sqrt{400 - 400}}{8} = \dfrac{20}{8} = 2.5$

d $x^2 - 5x + 7 = 0$

$x = \dfrac{5 \pm \sqrt{25 - 28}}{2}$ No solutions

e $x^2 + x - 18 = 0$

$x = \dfrac{-1 \pm \sqrt{1 + 72}}{2} = -4.77$ and 3.77

f $x^2 - 8 = 0$

$x = \dfrac{0 \pm \sqrt{0 + 32}}{2} = 2\sqrt{2} = -2.83$ and 2.83

3 a $x^2 - 2x - 2 = 0$

$x = \dfrac{2 \pm \sqrt{4 + 8}}{2} = -0.73$ and 2.73

b $x^2 - 4x - 11 = 0$

$x = \dfrac{4 \pm \sqrt{16 + 44}}{2} = -1.87$ and 5.87

c $x^2 - x - 5 = 0$

$x = \dfrac{1 \pm \sqrt{1 + 20}}{2} = -1.79$ and 2.79

d $x^2 + 2x - 7 = 0$

$x = \dfrac{-2 \pm \sqrt{4 + 28}}{2} = -3.83$ and 1.83

e $x^2 - 3x + 1 = 0$

$x = \dfrac{3 \pm \sqrt{9 - 4}}{2} = 0.38$ and 2.62

f $x^2 - 8x + 3 = 0$

$x = \dfrac{8 \pm \sqrt{64 - 12}}{2} = \dfrac{8 \pm \sqrt{52}}{2} = 0.39$ and 7.61

4 a $2x^2 - 3x - 4 = 0$

$x = \dfrac{3 \pm \sqrt{9 + 32}}{4} = -0.85$ and 2.35

b $4x^2 + 2x - 5 = 0$

$x = \dfrac{-2 \pm \sqrt{4 + 80}}{8} = -1.40$ and 0.90

c $5x^2 - 8x + 1 = 0$

$x = \dfrac{8 \pm \sqrt{64 - 20}}{10} = \dfrac{8 \pm \sqrt{44}}{10} = 0.14$ and 1.46

d $-2x^2 - 5x - 2 = 0$

$x = \dfrac{5 \pm \sqrt{25 - 16}}{-4} = \dfrac{5 \pm 3}{-4} = -2$ and -0.5

e $3x^2 - 4x - 2 = 0$

$x = \dfrac{4 \pm \sqrt{16 + 24}}{6} = \dfrac{4 \pm \sqrt{40}}{6} = -0.39$ and 1.72

f $-7x^2 - x + 15 = 0$

$x = \dfrac{1 \pm \sqrt{1 + 420}}{-14} = -1.54$ and 1.39

Exercise 6.4.3

1 Number of viruses after 24 hours $= 1(1 + 1)^{24}$
$= 2^{24} = 16\,777\,216$

2 We need to find x (the number of time intervals or half-lives) such that
$1 = 1000(1 - 0.5)^x$, i.e.
$\left(\frac{1}{2}\right)^x = 0.001$
Try $\left(\frac{1}{2}\right)^9 = 0.0019$
$\left(\frac{1}{2}\right)^{10} = 0.0009$
So it will take 10 time intervals, i.e.
$10 \times 24\,000 = 240\,000$ years.

3 Projected population in 2010 would have been
$650 \times (1.5)^5 = 4935$ million $= 4900$ million to 2 s.f.

4 Quantity remaining after 200 years
$= 100(0.5)^{10} = 0.098$ g

5 Entrants remaining after 6 rounds $= 512\left(\frac{1}{2}\right)^6 = 8$

6 a Area left after 20 years at 5% loss per year
$= 1\,000\,000 \times (0.95)^{20} = 358\,485$
$= 358\,000\,\text{km}^2$ to 3 s.f.

b We need to solve the equation:
$500\,000 = 1\,000\,000 \times (0.9)^x$, where x is the number of years
i.e. $(0.9)^x = 0.5$
$(0.9)^6 = 0.53$
$(0.9)^7 = 0.48$
So it will take 7 years for the area to cover less than 500 000.

7 $100\,000 = (1 + r)^{20}$, where r is the rate of increase
$1 + r = \sqrt[20]{1\,000\,000}$
$1 + r = 1.995$
So $r = 1.0$ (to 2 s.f.)

8 $\left(1 - \frac{x}{100}\right)^7 = 0.5$
$\left(1 - \frac{x}{100}\right) = \sqrt[7]{0.5} = 0.91$ (to 2 d.p.)
So $x = 9\%$ to the nearest whole number.

Student assessment 2 (Topic 6)

3 a $x^2 + 6x + 8 = 0$
$(x + 4)(x + 2) = 0$
$x = -4$ and $x = -2$

b $2x^2 + 10 = 12x$
$2x^2 - 12x + 10 = 0$
$x^2 - 6x + 5 = 0$
$(x - 5)(x - 1) = 0$
$x = 5$ and $x = 1$

c $x^2 + 10x + 25 = 0$
$(x + 5)(x + 5) = 0$
$x = -5$

d $3x^2 - 4 = 7x$
$3x^2 - 7x - 4 = 0$

4 a $4x^2 - 6x + 1 = 0$
$x = \dfrac{6 \pm \sqrt{36 - 16}}{8}$
$x = 0.191$ and 1.31

b $5x^2 - 12x - 3 = 0$
$x = \dfrac{12 \pm \sqrt{144 + 60}}{10}$
$x = 2.63$ and -0.228

5 Amount after 10 years $= 4000(1.075)^{10}$
$= €8244.13$

6 After 24 hours number of bacteria $= x(1.25)^{24}$
$= 211.8x$

7 After 21 years fraction remaining $= (0.9)^{21}$
$= 0.109$
After 22 years fraction remaining $= (0.9)^{22}$
$= 0.098$
So after 22 years 10 million barrels will be reduced to 1 million barrels (i.e. 0.1 of the original).

8 Fraction of light at 16 metres $= (0.875)^{16} = 0.12$
Fraction of light at 17 metres $= (0.875)^{17}$
$= 0.1033$
Fraction of light at 18 metres $= (0.875)^{18}$
$= 0.0904$
Fraction of light at 17.5 metres $= (0.875)^{17.5}$
$= 0.0966$
So fraction of light will be 0.1 (10%) between 17 and 17.5 metres, i.e. at 17 metres (to the nearest metre).

Exercise 7.1.3

1 a $f(x) = x^4$
$f'(x) = 4x^{4-1} = 4x^3$

b $f(x) = x^5$
$f'(x) = 5x^{5-1} = 5x^4$

c $f(x) = 3x^2$
$f'(x) = 2 \times 3x^{2-1} = 6x$

d $f(x) = 5x^3$
$f'(x) = 3 \times 5x^{3-1} = 15x^2$

e $f(x) = 6x^3$
$f'(x) = 3 \times 6x^{3-1} = 18x^2$

f $f(x) = 8x^7$
$f'(x) = 7 \times 8x^{7-1} = 56x^6$

2 a $f(x) = \frac{1}{3}x^3$
$f'(x) = 3 \times \frac{1}{3}x^{3-1} = x^2$

b $f(x) = \frac{1}{4}x^4$
$f'(x) = 4 \times \frac{1}{4}x^{4-1} = x^3$

c $f(x) = \frac{1}{4}x^2$
$f'(x) = 2 \times \frac{1}{4}x^{2-1} = \frac{1}{2}x$

d $f(x) = \frac{1}{2}x^4$
$f'(x) = 4 \times \frac{1}{2}x^{4-1} = 2x^3$

e $f(x) = \frac{2}{5}x^3$

$f'(x) = 3 \times \frac{2}{5}x^{3-1} = \frac{6}{5}x^2$

f $f(x) = \frac{2}{9}x^3$

$f'(x) = 3 \times \frac{2}{9}x^{3-1} = \frac{2}{3}x^2$

Exercise 7.2.1

1 a $y = 5x^3$

$\frac{dy}{dx} = 3 \times 5x^{3-1} = 15x^2$

b $y = 7x^2$

$\frac{dy}{dx} = 2 \times 7x^{2-1} = 14x$

c $y = 4x^6$

$\frac{dy}{dx} = 6 \times 4x^{6-1} = 24x^5$

d $y = \frac{1}{4}x^2$

$\frac{dy}{dx} = 2 \times \frac{1}{4}x^{2-1} = \frac{1}{2}x$

e $y = \frac{2}{3}x^6$

$\frac{dy}{dx} = 6 \times \frac{2}{3}x^{6-1} = 4x^5$

f $y = \frac{3}{4}x^5$

$\frac{dy}{dx} = 5 \times \frac{3}{4}x^{5-1} = \frac{15}{4}x^4$

g $y = 5 = 5x^0$

$\frac{dy}{dx} = 0 \times 5x^{0-1} = 0$

h $y = 6x$

$\frac{dy}{dx} = 6x^{1-1} = 6x^0 = 6$

i $y = \frac{1}{8} = \frac{1}{8}x^0$

$\frac{dy}{dx} = 0 \times \frac{1}{8}x^{0-1} = 0$

2 a $y = 3x^2 + 4x$

$\frac{dy}{dx} = 2 \times 3x^{2-1} + 4 \times x^{1-1}$

$= 6x + 4$

b $y = 5x^3 - 2x^2$

$\frac{dy}{dx} = 3 \times 5x^{3-1} - 2 \times 2x^{2-1}$

$= 15x^2 - 4x$

c $y = 10x^3 - \frac{1}{2}x^2$

$\frac{dy}{dx} = 3 \times 10x^{3-1} - 2 \times \frac{1}{2}x^{2-1}$

$= 30x^2 - x$

d $y = 6x^3 - 3x^2 + x$

$\frac{dy}{dx} = 3 \times 6x^{3-1} - 2 \times 3x^{2-1} + 1 \times x^{1-1}$

$= 18x^2 - 6x + 1$

e $y = 12x^4 - 2x^2 + 5$

$\frac{dy}{dx} = 4 \times 12x^{4-1} - 2 \times 2x^{2-1} + 0$

$= 48x^3 - 4x$

f $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 + x - 4$

$\frac{dy}{dx} = 3 \times \frac{1}{3}x^{3-1} - 2 \times \frac{1}{2}x^{2-1} + 1$

$= x^2 - x + 1$

g $y = -3x^4 + 4x^2 - 1$

$\frac{dy}{dx} = 4 \times -3x^{4-1} + 2 \times 4x^{2-1} + 0$

$= -12x^3 + 8x$

h $y = -6x^5 + 3x^4 - x + 1$

$\frac{dy}{dx} = 5 \times -6x^{5-1} + 4 \times 3x^{4-1} - 1 + 0$

$= -30x^4 + 12x^3 - 1$

i $y = -\frac{3}{4}x^6 + \frac{2}{3}x^3 - 8$

$\frac{dy}{dx} = 6 \times -\frac{3}{4}x^{6-1} + 3 \times \frac{2}{3}x^{3-1} + 0$

$= -\frac{9}{2}x^5 + 2x^2$

3 a $y = \frac{x^3 + x^2}{x} = x^2 + x$

$\frac{dy}{dx} = 2x^{2-1} + 1$

$= 2x + 1$

b $y = \frac{4x^3 - x^2}{x^2} = 4x - 1$

$\frac{dy}{dx} = 1 \times 4x^{1-1} - 0^2 = 4$

c $y = \frac{6x^3 + 2x^2}{2x} = 3x^2 + x$

$\frac{dy}{dx} = 2 \times 3x^{2-1} + 1$

$= 6x + 1$

d $y = \frac{x^3 + 2x^2}{4x} = \frac{x^2}{4} + \frac{x}{2}$

$\frac{dy}{dx} = \frac{2 \times x^{2-1}}{4} + \frac{1}{2}$

$= \frac{1}{2}x + \frac{1}{2}$

e $y = 3x(x + 1) = 3x^2 + 3x$

$\frac{dy}{dx} = 2 \times 3x^{2-1} + 3$

$= 6x + 3$

f $y = 2x^2(x - 2) = 2x^3 - 4x^2$

$\frac{dy}{dx} = 3 \times 2x^{3-1} - 2 \times 4x^{2-1}$

$= 6x^2 - 8x$

g $y = (x + 5)^2 = x^2 + 10x + 25$

$\frac{dy}{dx} = 2 \times x^{2-1} + 10 + 0$

$= 2x + 10$

h $y = (2x - 1)(x + 4) = 2x^2 + 7x - 4$

$\frac{dy}{dx} = 2 \times 2x^{2-1} + 7$

$= 4x + 7$

i $y = (x^2 + x)(x - 3) = x^3 - 2x^2 - 3x$

$\frac{dy}{dx} = 3 \times x^{3-1} - 2 \times 2x^{2-1} - 3$

$= 3x^2 - 4x - 3$

Exercise 7.2.2

1 a $y = x^{-1}$

$\frac{dy}{dx} = -1x^{-1-1} = -x^{-2}$

b $y = x^{-3}$

$\frac{dy}{dx} = -3x^{-3-1} = -3x^{-4}$

c $y = 2x^{-2}$

$\frac{dy}{dx} = -2 \times 2x^{-2-1} = -4x^{-3}$

d $y = -x^{-2}$

$\frac{dy}{dx} = -2 \times -x^{-2-1} = 2x^{-3}$

e $y = -\frac{1}{3}x^{-3}$

$\frac{dy}{dx} = -3 \times -\frac{1}{3}x^{-3-1} = x^{-4}$

f $y = -\frac{2}{5}x^{-5}$

$\frac{dy}{dx} = -5 \times -\frac{2}{5}x^{-5-1} = 2x^{-6}$

3 a $f(x) = 3x^{-1} + 2x$

$f'(x) = -1 \times 3x^{-1-1} + 2$

$= -3x^{-2} + 2$

b $f(x) = 2x^2 + x^{-1} + 1$

$f'(x) = 2 \times 2x^{2-1} - 1x^{-1-1} + 0$

$= 4x - x^{-2}$

c $f(x) = 3x^{-1} - x^{-2} + 2x$

$f'(x) = -3x^{-1-1} + 2x^{-2-1} + 2$

$= -3x^{-2} + 2x^{-3} + 2$

d $f(x) = \frac{1}{x^3} + x^3 = x^{-3} + x^3$

$f'(x) = -3 \times x^{-3-1} + 3x^{3-1}$

$= -3x^{-4} + 3x^2$

e $f(x) = \frac{2}{x^4} - \frac{1}{x^3} + 1 = 2x^{-4} - x^{-3} + 1$

$f'(x) = -4 \times 2x^{-5} + 3x^{-4} + 0$

$= -8x^{-5} + 3x^{-4}$

f $f(x) = \frac{1}{2x^2} + \frac{1}{3x^3} = \frac{1}{2}x^{-2} + \frac{1}{3}x^{-3}$

$f'(x) = -2 \times \frac{1}{2}x^{-2-1} - 3 \times \frac{1}{3}x^{-3-1}$

$= x^{-3} - x^{-4}$

Exercise 7.2.3

1 a $y = 3t^2 + t$

$\frac{dy}{dx} = 2 \times 3t^{2-1} + 1 = 6t + 1$

b $v = 2t^3 + t^2$

$\frac{dv}{dt} = 3 \times 2t^{3-1} + 2t^{2-1} = 6t^2 + 2$

c $m = 5t^3 - t^2$

$\frac{dm}{dt} = 3 \times 5t^{3-1} - 2t = 15t^2 - 2t$

d $y = 2t^{-1}$

$\frac{dy}{dt} = -1 \times 2t^{-1-1} = -2t^{-2}$

e $r = \frac{1}{2}t^{-2}$

$\frac{dr}{dt} = -2 \times \frac{1}{2}t^{-2-1} = -t^{-3}$

f $s = t^4 - t^{-2}$

$\frac{ds}{dt} = 4 \times t^{4-1} + 2t^{-3} = 4t^3 + 2t^{-3}$

2 a $y = 3x^{-1} + 4$

$\frac{dy}{dx} = -1 \times 3x^{-1-1} = -3x^{-2}$

b $s = 2t^{-1} - t$

$\frac{ds}{dt} = -2t^{-1-1} - 1 = 2t^{-2} - 1$

c $v = r^{-2} - \frac{1}{r}$

$\frac{dv}{dr} = -2r^{-3} - 1 \times -1r^{-1-1} = -2r^{-3} + r^{-2}$

d $P = \frac{l^{-4}}{2} + 2l$

$\frac{dP}{dl} = -2l^{-5} + 2$

e $m = \frac{n}{2} - \frac{n^{-3}}{3}$

$\frac{dm}{dn} = \frac{1}{2} + n^{-3-1} = \frac{1}{2} + n^{-4}$

f $a = \frac{2t^{-2}}{5} - t^3$

$\frac{da}{dt} = -\frac{4t^{-3}}{5} - 3t^2$

3 a $y = x(x+4) = x^2 + 4x$

$\frac{dy}{dx} = 2x + 4$

b $r = t(1-t) = t - t^2$

$\frac{dr}{dt} = 1 - 2t$

c $v = t\left(\frac{1}{t} + t^2\right) = 1 + t^3$

$\frac{dv}{dt} = 3t^2$

d $p = r^2\left(\frac{2}{r} - 3\right) = 2r - 3r^2$

e $a = x\left(x^{-2} + \frac{x}{2}\right) = x^{-1} + \frac{x^2}{2}$

$\frac{da}{dx} = -x^{-2} + x$

f $y = t^{-1}\left(t - \frac{1}{t^2}\right) = 1 - t^{-3}$

$\frac{dy}{dt} = 3t^{-4}$

4 a $y = (t+1)(t-1) = t^2 - 1$

$\frac{dy}{dt} = 2t$

b $r = (t-1)(2t+2) = 2t^2 - 2$

$\frac{dr}{dt} = 4t$

c $p = \left(\frac{1}{t} + 1\right)\left(\frac{1}{t} - 1\right) = \frac{1}{t^2} - 1 = t^{-2}$

$\frac{dp}{dt} = -2t^{-3}$

d $a = (t^{-2} + t)(t^2 - 2) = 1 - 2t^{-2} + t^3 - 2t$

$\frac{da}{dt} = 3t^2 - 2 + 4t^{-3}$

e $v = \left(\frac{2t^2}{3} + 1\right)(t-1) = \frac{2t^3}{3} - \frac{2t^2}{3} + t - 1$

$\frac{dv}{dt} = 2t^2 - \frac{4}{3}t + 1$

f $y = \left(\frac{3}{2t^4} - t\right)\left(2t - \frac{3}{t}\right) = 3t^{-3} - \frac{9}{2}t^{-5} - 2t^2 + 3$

$\frac{dy}{dt} = -9t^{-4} + \frac{45}{2}t^{-6} - 4t$

Exercise 7.3.1

1 a $f(x) = x^2$
$f'(x) = 2x$
$f'(3) = 2 \times 3 = 6$

b $f(x) = \frac{1}{2}x^2 - 2$
$f'(x) = x$
$f'(-3) = -3$

c $f(x) = 3x^3 - 4x^2 - 2$
$f'(x) = 9x^2 - 8x$
$f'(0) = 0$

d $f(x) = -x^2 + 2x - 1$
$f'(x) = -2x + 2$
$f'(1) = -2 + 2 = 0$

e $f(x) = -\frac{1}{2}x^3 + x - 3$

$f'(x) = -\frac{3}{2}x^2 + 1$

$f'(-1) = -\frac{3}{2} + 1 = -\frac{1}{2}$

$f'(2) = -\frac{3}{2} \times 4 + 1 = -6 + 1 = -5$

f $f(x) = 6x$
$f'(x) = 6$
$f'(3) = 6$

2 a $f(x) = \frac{1}{x} = x^{-1}$
$f'(x) = -x^{-2}$
$f'(2) = -\frac{1}{4}$

b $f(x) = \frac{1}{x^2} = x^{-2}$
$f'(x) = -2x^{-3}$
$f'(1) = -2$

c $f(x) = \frac{1}{x^3} - 3x = x^{-3} - 3x$
$f'(x) = -3x^{-4} - 3$
$f'(2) = \frac{-3}{2^4} - 3 = -3\frac{3}{16}$

d $f(x) = x^2 - \frac{1}{2x^2} = x^2 - \frac{1}{2}x^{-2}$
$f'(x) = 2x + x^{-3}$
$f'(-1) = -2 - 1 = -3$

e $f(x) = \frac{1}{6x^3} + x^2 - 1$

$f'(x) = -\frac{1}{2}x^{-4} + 2x$

$f'(2) = -\frac{1}{2} \times \frac{1}{16} + 4 = 3\frac{31}{32}$

f $f(x) = \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} = x^{-1} - x^{-2} + x^{-3}$

$f'(x) = -x^{-2} + 2x^{-3} - 3x^{-4}$

$f'\left(\frac{1}{2}\right) = -4 + 16 - 48 = -36$

$f'\left(-\frac{1}{2}\right) = -4 - 16 - 48 = -68$

3 $N = 5t^2 - \frac{1}{2}t^3$

a i $N(1) = 5 - \frac{1}{2} = 4\frac{1}{2}$

ii $N(3) = 5 \times 9 - \frac{1}{2} \times 27 = 31\frac{1}{2}$

iii $N(6) = 5 \times 36 - \frac{1}{2} \times 216 = 72$

iv $N(10) = 5 \times 100 - \frac{1}{2} \times 1000 = 0$

b $\frac{dN}{dt} = 10t - \frac{3}{2}t^2$

c i When $t = 1$, $\frac{dN}{dt} = 10 - \frac{3}{2} = 8\frac{1}{2}$

ii When $t = 3$, $\frac{dN}{dt} = 30 - \frac{3}{2} \times 9 = 16\frac{1}{2}$

iii When $t = 6$, $\frac{dN}{dt} = 60 - \frac{3}{2} \times 36 = 6$

iv When $t = 10$, $\frac{dN}{dt} = 100 - \frac{3}{2} \times 100 = -50$

4 $h = 30t^2 - t^3$

a i $h(3) = (30 \times 9) - 27 = 243\,\text{m}$

ii $h(10) = (30 \times 100) - 1000 = 2000\,\text{m}$

b $\frac{dh}{dt} = 60t - 3t^2$

c i When $t = 2$, $\frac{dh}{dt} = 120 - 12 = 108\,\text{m h}^{-1}$

ii When $t = 5$, $\frac{dh}{dt} = 300 - 75 = 225\,\text{m h}^{-1}$

iii When $t = 20$, $\frac{dh}{dt} = 1200 - 1200 = 0\,\text{m h}^{-1}$

Exercise 7.3.2

3 a $s = 4t + 5t^2$

$\frac{ds}{dt} = 4 + 10t$

b $9 = 4 + 10t$ So $10t = 5$, i.e. $t = \frac{1}{2}$ seconds

c When stone hits ground $34 = 4 + 10t$

$10t = 30$, i.e. $t = 3$ seconds

d Height of cliff is distance travelled in 3 seconds,

$s(3) = (4 \times 3) + (5 \times 3^2) = 57\,\text{m}$

4 a $T = 20 + 12t^2 - t^3$

$T(0) = 20°\text{C}$

b $\frac{dT}{dt} = 24t - 3t^2$

c i When $t = 1$, $\frac{dT}{dt} = 24 - 3°\text{C/min}$

ii When $t = 4$, $\frac{dT}{dt} = (24 \times 4) - (3 \times 4^2)$

$= 48°\text{C/min}$

iii When $t = 8$, $\frac{dT}{dt} = (24 \times 8) - (3 \times 8^2)$

$= 0°\text{C/min}$

d $36 = 24t - 3t^2$

$3t^2 - 24t + 36 = 0$

$t^2 - 8t + 12 = 0$

$(t - 6)(t - 2) = 0$

So cooker could have been switched off at $t = 2$ or 6 minutes.

e $T(6) = 20 + (12 \times 36) - 216 = 236°\text{C}$

Exercise 7.3.3

1 a $f(x) = x^2 - 3x + 1$

$f'(x) = 2x^{2-1} - 3 = 2x - 3$

b At (2, 1) $f'(x) = 2 \times 2 - 3 = 1$

c Gradient of tangent at A is gradient of curve = 1 (as in b)

d Tangent passes through A(2, 1)

So $1 = 1 \times 2 + c$, giving $c = -1$

Equation of tangent is therefore $y = x - 1$

e Gradient of normal × gradient of tangent = −1

So gradient of normal = −1

f Normal passes through A(2, 1)

So $1 = -1 \times 2 + c$, giving $c = 3$

Equation of normal is $y = -x + 3$

2 a $f(x) = 2x^2 - 4x - 2$

$f'(x) = 4x - 4$

$f'(2) = 8 - 4 = 4$

b Tangent has gradient 4 and passes through (2, −2)

So $-2 = 4 \times 2 + c$, giving $c = -10$

So equation of tangent is $y = 4x - 10$

c Gradient of normal × gradient of tangent = −1

So gradient of normal $= -\frac{1}{4}$

d Normal passes through (2, –2)
So $-2 = -\frac{1}{4} \times 2 + c$, giving $c = -\frac{3}{2}$
Equation of normal is $y = -\frac{1}{4}x - \frac{3}{2}$
or $x + 4y + 6 = 0$

3 a $f(x) = \frac{1}{2}x^2 - 4x - 2$
$f'(x) = x - 4$
Gradient at P(0, –2) is
$f'(0) = -4$

b Tangent passes through (0, –2)
So $-2 = -4 \times 0 + c$, giving $c = -2$
Equation of tangent is $y = -4x - 2$

c Gradient of normal × gradient of tangent = –1
So gradient of normal at P = $\frac{-1}{-4} = \frac{1}{4}$
Normal passes through (0, –2)
So $-2 = \frac{1}{4} \times 0 + c$, giving $c = -2$
Equation of normal is $y = \frac{1}{4}x - 2$

4 a $f(x) = -\frac{1}{4}x^2 - 3x - 2$
$f'(x) = -\frac{1}{2}x - 3$
$f'(-2) = \left(-\frac{1}{2} \times -2\right) - 3 = -2$
So gradient of $T_1 = -2$
T_1 passes through P(–2, 6)
So $6 = -2 \times -2 + c$, giving $c = 2$
Equation of T_1 is $y = -2x + 2$

b T_2 has equation $y = 10$ and gradient $m = 0$
At Q $f'(x) = -\frac{1}{2}x - 3 = 0$, i.e. $x = -6$
So Q is (–6, 10)

c T_1 and T_2 intersect where $10 = -2x + 2$, i.e. at $x = -4$, at the point (–4, 10)

5 a $f(x) = -x^2 + 4x + 1$
$f'(x) = -2x + 4$
At A(4, 1)
$f'(4) = -8 + 4 = -4$

b Tangent passes through (4, 1)
So $1 = -4 \times 4 + c$, giving $c = 17$
Equation of tangent is $y = -4x + 17$

c At (2, 5) gradient of tangent T_2 is
$f'(2) = (-2 \times 2) + 4 = 0$ and
$5 = 0 + c$
So equation of T_2 is $y = 5$

d At A gradient of normal is $\frac{-1}{-4} = \frac{1}{4}$
Normal passes through (4, 1)
So $1 = 4 \times \frac{1}{4} + c$, giving $c = 0$
So N_1 has equation $y = \frac{1}{4}x$
At B the tangent is horizontal so the normal will be a vertical line passing through (2, 5).
So equation of normal N_2 at B is $x = 2$

e N_1 and N_2 intersect where $y = \frac{1}{4} \times 2 = \frac{1}{2}$,
i.e. at $\left(2, \frac{1}{2}\right)$

6 a $f(x) = -\frac{1}{2}x^2 - x - 4$
$f'(x) = -x - 1$

b T has equation $y = -3x - 6$
Gradient is $\frac{dy}{dx} = -3$

c At P gradients are equal, so
$-x - 1 = -3$
$x = 2$
Substituting into equation for the curve
$y = -8$
So P is (2, –8)

Exercise 7.4.1

1 a i $f(x) = x^2 - 4$
$f'(x) = 2x$

ii $f(x)$ is increasing when $f'(x) > 0$, i.e. when $x > 0$

b i $f(x) = x^2 - 3x + 10$
$f'(x) = 2x - 3$

ii $f'(x) > 0$ when $2x - 3 > 0$, i.e. when $x > \frac{3}{2}$

c i $f(x) = -x^2 + 10x - 21$
$f'(x) = -2x + 10$

ii $f'(x) > 0$ when $-2x + 10 > 0$, i.e. when $x < 5$

d i $f(x) = x^3 - 12x^2 + 48x - 62$
$f'(x) = 3x^2 - 24x + 48$

ii $f'(x) > 0$ when $3x^2 - 24x + 48 > 0$
$x^2 - 8x + 16 > 0$
$(x - 4)^2 > 0$, i.e. when $x < 4$, $x > 4$

e i $f(x) = -x^3 + 25x$
$f'(x) = -3x^2 + 25$

ii $f'(x) > 0$ when $-3x^2 - 25 > 0$
$x^2 < \frac{25}{3}$, i.e. $-\frac{5}{\sqrt{3}} < x < \frac{5}{\sqrt{3}}$

f i $f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2$
$f'(x) = x^3 - x$

ii $f'(x) > 0$ when $x^2 - x > 0$
$x(x^2 - 1) > 0$
If $x > 0$, $f'(x) > 0$ when $x > 1$
If $x < 0$, $f'(x) > 0$ when $-1 < x < 0$

2 a Decreasing when $x < 0$
b Decreasing when $x < \frac{3}{2}$
c Decreasing when $x < 5$
d Increasing everywhere but at $x = 4$ where $f'(x) = 0$. Decreases nowhere.
e Decreases when $x < -\dfrac{5}{\sqrt{3}}$, $x > \dfrac{5}{\sqrt{3}}$
f When $x > 0$ decreases when $0 < x < 1$
When $x < 0$ decreases when $x < -1$

4 $f(x) = x^3 + x^2 - kx$
$f'(x) = 3x^2 + 2x - k$
If $f(x) > 0$ for all x,
$3x^2 + 2x - k > 0$
$3\left(x + \frac{1}{3}\right)^2 - \frac{1}{3} - k > 0$
The first term is always positive (or zero) so $f(x) > 0$ when $-\frac{1}{3} - k > 0$, i.e. when $k < -\frac{1}{3}$

Exercise 7.5.1

1 a i $f(x) = x^2 - 6x + 13$
$f'(x) = 2x - 6$
ii Stationary points where $f'(x) = 0$,
i.e. where
$2x - 6 = 0$
$x = 3$
If $x = 3$, $y = 9 - 18 + 13 = 4$
Stationary point at (3, 4)

b i $f(x) = x^2 + 12x + 35$
$f'(x) = 2x + 12$
ii Stationary points where $f'(x) = 0$,
i.e. where
$2x + 12 = 0$
$x = -6$
If $x = -6$, $y = 36 - 72 + 35 = -1$
Stationary point at (–6, –1)

c i $f(x) = -x^2 + 8x - 13$
$f'(x) = -2x + 8$
ii Stationary points where $f'(x) = 0$,
i.e. where
$-2x + 8 = 0$
$x = 4$
If $x = 4$, $y = -16 + 32 - 13 = 3$
Stationary point at (4, 3)

d i $f(x) = -6x + 7$
$f'(x) = -6$
ii $f'(x) = 0$ nowhere; so no stationary points

2 a i $f(x) = x^3 - 12x^2 + 48x - 58$
$f'(x) = 3x^2 - 24x + 48$
ii Stationary points where $f'(x) = 0$,
i.e. where
$3x^2 - 24x + 48 = 0$
$x^2 - 8x + 16 = 0$
$(x - 4)^2 = 0$
$x = 4$
If $x = 4$,
$y = 64 - (12 \times 16) + (48 \times 4) - 58 = 6$
Stationary point at (4, 6)

b i $f(x) = x^3 - 12x$
$f'(x) = 3x^2 - 12$
ii Stationary points where $f'(x) = 0$,
i.e. where
$3x^2 - 12 = 0$
$x^2 - 4 = 0$
$(x + 2)(x - 2) = 0$
At $x = 2$ and $x = -2$
When $x = 2$, $y = 8 - 24 = -16$
When $x = -2$, $y = -8 + 24 = 16$
Stationary points at (2, –16) and (–2, 16)

c i $f(x) = x^3 - 3x^2 - 45x + 8$
$f'(x) = 3x^2 - 6x - 45$
ii Stationary points where $f'(x) = 0$,
i.e. where
$3x^2 - 6x - 45 = 0$
$x^2 - 2x - 15 = 0$
$(x - 5)(x + 3) = 0$
At $x = 5$ and $x = -3$
When $x = 5$,
$y = 5^3 - (3 \times 5^2) - (45 \times 5) + 8 = -167$
When $x = -3$,
$y = (-3)^3 - (3 \times 9) - (45 \times -3) + 8 = 89$
Stationary points at (5, –167) and (–3, 89)

d i $f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 4x - 5$
$f'(x) = x^2 + 3x - 4$
ii Stationary points where $f'(x) = 0$,
i.e. where
$x^2 + 3x - 4 = 0$
$(x + 4)(x - 1) = 0$
At $x = -4$ and $x = 1$
When $x = -4$,
$y = \frac{1}{3}(-4)^3 + \frac{3}{2}(-4)^2 + 16 - 5 = \frac{41}{3} = 13\frac{2}{3}$

When $x = 1$, $y = \frac{1}{3} + \frac{3}{2} - 4 - 5 = -7\frac{1}{6}$

Stationary points at $\left(-4, 13\frac{2}{3}\right)$ and $\left(1, -7\frac{1}{6}\right)$

3 a i $f(x) = 1 - 4x - x^2$

$f'(x) = -4 - 2x$

ii Stationary points where $f'(x) = 0$, i.e. where

$-4 - 2x = 0$

$x = -2$

At $x = -2$, $y = 1 + 8 - 4 = 5$

Stationary point at $(-2, 5)$

iii Near $x = -2$

$f'(-3) = -4 + 6 = 2$

$f'(-1) = -4 + 2 = -2$

Gradient changes from positive to negative as x increases so $(-2, 5)$ is a maximum.

iv $f(0) = 1$

b i $f(x) = \frac{1}{3}x^3 - 4x^2 + 12x - 3$

$f'(x) = x^2 - 8x + 12$

ii Stationary points where $f'(x) = 0$, i.e. where

$x^2 - 8x + 12 = 0$

$(x - 6)(x - 2) = 0$

$x = 6$ and $x = 2$

When $x = 6$,

$y = \left(\frac{1}{3} \times 216\right) - \left(4 \times 36\right) + \left(12 \times 6\right) - 3 = -3$

When $x = 2$, $y = \left(\frac{1}{3} \times 8\right) - 16 + 24 - 3 = 7\frac{2}{3}$

Stationary points at $(6, -3)$ and $\left(2, 7\frac{2}{3}\right)$

iii Near $x = 2$

$f'(1) = 1 - 8 + 12 = 5$

$f'(3) = 9 - 24 + 12 = -3$

Gradient changes from positive to negative so $\left(2, 7\frac{2}{3}\right)$ is a maximum.

Near $x = 6$

$f'(5) = 25 - 40 + 12 = -3$

$f'(7) = 49 - 56 + 12 = 5$

Gradient changes from negative to positive so $(6, -3)$ is a minimum.

iv $f(0) = -3$

c i $f'(x) = -\frac{2}{3}x^3 + 3x^2 - 4x$

$f'(x) = -2x^2 + 6x - 4$

ii Stationary points where $f'(x) = 0$, i.e. where

$-2x^2 + 6x - 4 = 0$

$x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0$

At $x = 2$ and $x = 1$

When $x = 2$, $y = \left(-\frac{2}{3} \times 8\right) + 12 - 8 = -1\frac{1}{3}$

When $x = 1$, $y = -\frac{2}{3} + 3 - 4 = -1\frac{2}{3}$

Stationary points $\left(2, -1\frac{1}{3}\right)$ and $\left(1, -1\frac{2}{3}\right)$

iii Near $x = 2$

$f'(1.5) = -2 \times 1.5^2 + 9 - 4 = 0.5$

$f'(3) = -18 + 18 - 4 = -4$

Gradient changes from positive to negative as x increases so $\left(2, -1\frac{1}{3}\right)$ is a maximum.

Near $x = 1$

$f'(0.5) = -2 \times 0.5^2 + 3 - 4 = -1.5$

$f'(1.5) = 0.5$ (from above)

Gradient changes from negative to positive as x increases so $\left(1, -1\frac{2}{3}\right)$ is a minimum.

iv $f(0) = 0$

d i $f(x) = x^3 - \frac{9}{2}x^2 - 30x + 4$

$f'(x) = 3x^2 - 9x - 30$

ii Stationary points where $f'(x) = 0$, i.e. where

$3x^2 - 9x - 30 = 0$

$x^2 - 3x - 10 + 0$

$(x - 5)(x + 2) = 0$

At $x = 5$ and $x = -2$

When $x = 5$,

$y = 125 - \left(\frac{9}{2} \times 25\right) - 150 + 4 = -133\frac{1}{2}$

When $x = -2$,

$y = -8 - \left(\frac{9}{2} \times 4\right) + 60 + 4 = 38$

Stationary points are $\left(5, -133\frac{1}{2}\right)$ and $(-2, 38)$

iii Near $x = 5$

$f'(4) = 48 - 36 - 30 = -18$

$f'(6) = 108 - 45 - 30 = 33$

Gradient changes from negative to positive as x increases so $\left(5, -133\frac{1}{2}\right)$ is a minimum.

Near $x = -2$

$f'(-3) = 27 + 27 - 30 = 24$

$f'(-1) = 3 + 9 - 30 = -18$

Gradient changes from positive to negative so $(-2, 38)$ is a maximum.

iv $f(0) = 4$

4 a i $f(x) = x^3 - 9x^2 + 27x - 30$
$f'(x) = 3x^2 - 18x + 27$

ii Stationary points where $f'(x) = 0$, i.e. where
$3x^2 - 18x + 27 = 0$
$x^2 - 6x + 9 = 0$
$(x - 3)^2 = 0$
At $x = 3$
When $x = 3$, $y = 27 - 81 + 81 - 30 = -3$
Stationary point at $(3, -3)$

iii Near $x = 3$
$f'(2) = 12 - 36 + 27 = 3$
$f'(4) = 48 - 72 + 27 = 3$
As $f'(x > 0)$ above and below $x = 3$, $(3, -3)$ is a point of inflexion.

iv $f(0) = -30$

b i $f(x) = x^4 - 4x^3 + 16x$
$f'(x) = 4x^3 - 12x^2 + 16$

ii Stationary points where $f'(x) = 0$, i.e. where
$4x^3 - 12x^2 + 16 = 0$
$x^3 - 3x^2 + 4 = 0$
$(x + 1)(x^2 - 4x + 4) = 0$
$(x + 1)(x - 2)^2 = 0$
At $x = -1$ and $x = 2$
When $x = -1$, $y = 1 + 4 - 16 = -11$
When $x = 2$, $y = 16 - 32 + 32 = 16$
Stationary points at $(-1, -11)$ and $(2, 16)$

iii Near $x = -1$
$f'(-2) = -32 - 48 + 16 = -64$
$f'(0) = 16$
Gradient changes from negative to positive as x increases so $(-1, -11)$ is a minimum.
Near $x = 2$
$f'(1) = 4 - 12 + 16 = 8$
$f'(3) = (4 \times 27) - (12 \times 9) + 16 = 16$
As $f'(x) > 0$ above and below $x = 2$, $(2, 16)$ is a point of inflexion.

iv $f(0) = 0$

Student assessment 1 (Topic 7)

1 a $y = x^3, \dfrac{dy}{dx} = 3x^{3-1} = 3x^2$

b $y = 2x^2 - x, \dfrac{dy}{dx} = 2 \times 2x^{2-1} - 1 = 4x - 1$

c $y = -\frac{1}{2}x^2 + 2x, \dfrac{dy}{dx} = 2 \times -\frac{1}{2}x^{2-1} + 2 = -x + 2$

d $y = \frac{2}{3}x^3 + 4x^2 - x, \dfrac{dy}{dx} = 3 \times \frac{2}{3}x^{3-1} + 8x - 1$
$= 2x^2 + 8x - 1$

2 a $f(x) = x(x + 2) = x^2 + 2x$
$f'(x) = 2x + 2$

b $f(x) = (x + 2)(x - 3) = x^2 - x - 6$
$f'(x) = 2x - 1$

c $f(x) = \dfrac{x^3 - x}{x} = x^2 - 1$
$f'(x) = 2x$

d $f(x) = \dfrac{x^3 + 2x^2}{2x} = \dfrac{x^2}{2} + x$
$f'(x) = x + 1$

e $f(x) = \dfrac{3}{x} = 3x^{-1}$
$f'(x) = -1 \times 3x^{-1-1} = -3x^{-2}$

f $f(x) = \dfrac{x^2 + 2}{x} = x + 2x^{-1}$
$f'(x) = 1 - 2x^{-2}$

3 a $f(x) = \frac{1}{2}x^2 + x$
$f'(x) = x + 1$
At $x = 1$, $f'(x) = 2$

b $f(x) = -x^3 + 2x^2 + x$
$f'(x) = -3x^2 + 4x + 1$
So $f'(0) = 1$

c $f(x) = \dfrac{1}{2x^2} + x$
$f'(x) = -x^{-3} + 1$
So $f'\left(-\frac{1}{2}\right) = 8 + 1 = 9$

d $f(x) = (x - 3)(x + 8) = x^2 + 5x - 24$
$f'(x) = 2x + 5$
So $f'\left(\frac{1}{4}\right) = 5\frac{1}{2}$

4 gradient normal × gradient tangent = −1

a gradient normal $= -\frac{1}{2}$

b gradient normal $= -\frac{1}{1} = -1$

c gradient normal $= -\frac{1}{9}$

d gradient normal $= -\dfrac{\frac{1}{11}}{2} = -\dfrac{2}{11}$

5 a $s = 5t^2$
$v = \dfrac{ds}{dt} = 10t$

b After 3 seconds $v = 10 \times 3 = 30\,\text{ms}^{-1}$

c **i** $42 = 10t$
So stone hits the ground at $t = 4.2$ s
ii Height of cliff, $s = 5t^2 = 5 \times 4.2^2 = 88.2$ m

Student assessment 2 (Topic 7)

1 **a** $f(x) = x^3 + x^2 - 1$
$f'(x) = 3x^{3-1} + 2x^{2-1} = 3x^2 + 2x$
b $f'(x) = 0$ when $3x^2 + 2x = 0$,
i.e., $x(3x + 2) = 0$
At $x = 0$ and $x = -\frac{2}{3}$
When $x = -\frac{2}{3}$
$y = -\frac{8}{27} + \frac{4}{9} - 1 = -\frac{23}{27}$
So P is $\left(-\frac{2}{3}, -\frac{23}{27}\right)$
c When $x = 0$, $y = -1$
So Q is $(0, -1)$
d Near P, consider the gradient at $x = -1$ and $x = -\frac{1}{2}$:
$f'(-1) = 3 - 2 = 1$
$f'\left(-\frac{1}{2}\right) = \frac{3}{4} - 1 = -\frac{1}{4}$
Gradient changes from positive to negative as x increases, so P must be a maximum.
Near Q, consider the gradient at $x = -\frac{1}{2}$ and $x = 1$:
$f'\left(-\frac{1}{2}\right) = \frac{3}{4} - 1 = -\frac{1}{4}$
$f'(1) = 5$
Gradient changes from negative to positive as x increases, so Q must be a minimum.

3 **a** $f(x) = (x - 2)^2 + 3 = x^2 - 4x + 7$
$f'(x) = 2x - 4$
b $f(x)$ is decreasing when $f'(x) < 0$
$2x - 4 < 0$
$2x < 4$
$x < 2$

4 **a** $f(x) = x^4 - 2x^2$
$f'(x) = 4x^3 - 4x$
b Stationary points when $f'(x) = 0$
$4x^3 - 4x = 4x(x^2 - 1) = 4x(x + 1)(x - 1) = 0$
At $x = 0$ and $x = \pm 1$
At $x = 0$, $y = 0$, i.e. $(0, 0)$
At $x = 1$, $y = -1$, i.e. $(1, -1)$
At $x = -1$, $y = -1$, i.e. $(-1, -1)$
c Near $x = -1$, consider the gradients at $x = -\frac{3}{2}$ and $x = -\frac{1}{2}$
$f'\left(-\frac{3}{2}\right) < 0$
$f'\left(-\frac{1}{2}\right) > 0$
Gradient changes from negative to positive as x increases so $(-1, -1)$ is a minimum.
Near $x = 0$, consider the gradients at $x = -\frac{1}{2}$ and $x = \frac{1}{2}$
$f'\left(\frac{1}{2}\right) < 0$
Gradient changes from positive to negative as x increases so $(0, 0)$ is a maximum.
Near $x = 1$, consider the gradients at $x = \frac{1}{2}$ and $x = \frac{3}{2}$
$f'\left(\frac{3}{2}\right) > 0$
Gradient changes from negative to positive as x increases so $(1, -1)$ is a minimum.
d **i** Graph intersects y-axis when $x = 0$
$f(0) = 0$, i.e. at $(0, 0)$
ii Graph intersects x-axis when $x^4 - 2x^2 = 0$
$x^2(x^2 - 2) = 0$
i.e. when $x = 0$, $x = \pm\sqrt{2}$ at the points
$(0, 0)$, $(\sqrt{2}, 0)$, $(-\sqrt{2}, 0)$